MTH 251 – Test 2
Given November 4, 2015

Name Key

To earn full credit on any given problem your work must be presented in a manner consistent with that demonstrated and discussed during both lecture and lab. This includes, but is no limited to, showing the Leibniz steps for every application of the product rule and quotient rule.

1. Find the first derivative formula for each of the following functions. In each case take the derivative with respect to x. Make sure that you use the appropriate name for each derivative.

   Please note that you will not receive full credit if you apply either the product rule or the quotient rule in any of these problems. (10 points total)

a. $f(x)=\dfrac{5\ln(x)}{6}$

$$f'(x) = \frac{5}{6}\cdot\frac{1}{x} = \frac{5}{6x}$$

b. $y=-\dfrac{1}{5x}$

$$y = -\frac{1}{5}x^{-1}$$

$$\frac{dy}{dx} = \frac{1}{5}x^{-2} = \frac{1}{5x^2}$$

c. $T=18\sqrt[3]{x^5}$

$$T = 18x^{5/3}$$

$$\frac{dT}{dx} = 30x^{2/3} = 30\sqrt[3]{x^2}$$

d. $z(x)=\pi^x+x^\pi$

$$z'(x) = \ln(\pi)\cdot\pi^x + \pi x^{\pi-1}$$

2. Let $S=2\pi r h+\pi r^2$. Find $\dfrac{dS}{dr}$ treating h as a constant. (3 points)

$$\frac{dS}{dr} = 2\pi h + 2\pi r$$

3. Find the first derivative formula for each of the following functions. In each case take the derivative with respect to the independent variable as implied by the expression on the right side of the equal sign. Make sure that you use the appropriate name for each derivative. This problem continues on page 3.

Please note that you will not receive full credit if you do not show the Leibniz step each and every time you apply either the product rule or the quotient rule. (20 points total)

a. $h(x)=\dfrac{3e^x}{7-e^x}$

$$h'(x) = \frac{\frac{d}{dx}(3e^x)\cdot(7-e^x) - (3e^x)\cdot\frac{d}{dx}(7-e^x)}{(7-e^x)^2}$$

$$= \frac{3e^x(7-e^x) - 3e^x\cdot -e^x}{(7-e^x)^2}$$

$$= \frac{21e^x - 3e^{2x} + 3e^{2x}}{(7-e^x)^2}$$

$$= \frac{21e^x}{(7-e^x)^2}$$

b. $y = x\ln(x)\cos(x)$

$$\frac{dy}{dx} = \frac{d}{dx}(x)\cdot \ln(x)\cos(x) + x\cdot\frac{d}{dx}(\ln(x))\cdot\cos(x) + x\ln(x)\cdot\frac{d}{dx}(\cos(x))$$

$$= 1\cdot\ln(x)\cos(x) + x\cdot\frac{1}{x}\cdot\cos(x) + x\ln(x)\cdot -\sin(x)$$

$$= \ln(x)\cos(x) + \cos(x) - x\ln(x)\sin(x)$$

c. $k = 4\sin^2(t)$ $k = 4\sin(t)\sin(t)$

$$\frac{dk}{dt} = \frac{d}{dt}(4\sin(t))\sin(t) + 4\sin(t)\cdot\frac{d}{dt}(\sin(t))$$

$$= 4\cos(t)\sin(t) + 4\sin(t)\cos(t)$$

$$= 8\sin(t)\cos(t)$$

d. $p = 2\ln(5) + x^2\tan(x)$

$$\frac{dp}{dt} = 0 + \frac{d}{dx}(x^2)\tan(x) + x^2\cdot\frac{d}{dx}(\tan(x))$$

$$= 2x\tan(x) + x^2\sec^2(x)$$

4. A function g is shown in Figure 1. The function has inflection points at -4, -2, 0, 2, and 4. Sketch onto Figure 2 a graph of g'. (10 points)

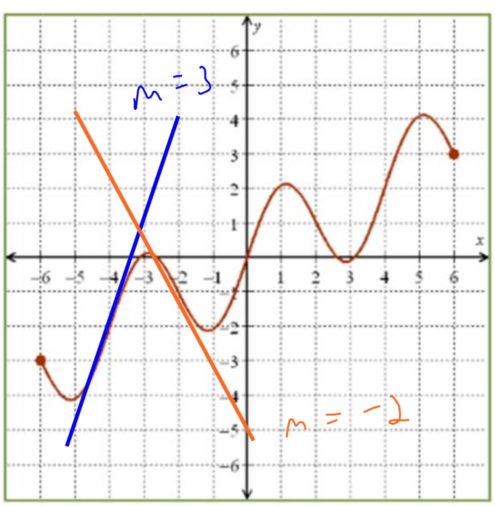


Figure 1: g

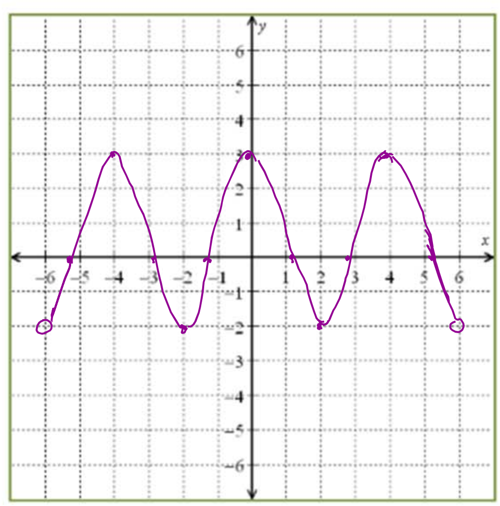


Figure 2: g'

5. Suppose that $S(x)$ is the average depth of the snowpack on Mt. Hood (measured in feet) at an elevation of x meters

   a. What is the unit for values of $S'(x)$? (2 points)

   The unit for values of $S'(x)$ is ft/m

   b. Suppose that $S'(x) = .02$ (unit from 5a) over the entire interval $(2000, 2100)$. What would this tell you about the snowpack on Mt. Hood? (5 points)

   Between the elevations of 2000 m and 2100m, the average depth of the snow pack on Mt. Hood increases by a total of 20ft.

   $(.02 \text{ ft/m})(1000\text{m}) = 20\text{ft}$

6. Find the first derivative with respect to x for the function $f(x) = \dfrac{x^2 - 1}{\sqrt{x}}$ and then find the equation of the tangent line at the point where $x = 1$. Make sure that your reasoning and conclusion are both clear (that means writing some relevant words).

   Please note that you will not receive full credit if you apply either the quotient rule or the product rule; you should simplify the formula before taking the derivative. (9 points)

$$f(x) = \frac{x^2-1}{\sqrt{x}} = \frac{x^2}{\sqrt{x}} - \frac{1}{\sqrt{x}} = x^{3/2} - x^{-1/2}$$

$$f'(x) = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-3/2} = \frac{3\sqrt{x}}{2} + \frac{1}{2\sqrt{x^3}}$$

$f(1) = 0$

$\therefore$ A point on the line is $(1,0)$

$f'(1) = \frac{3}{2} + \frac{1}{2} = 2$

$\therefore$ The slope of the line is 2.

$\therefore$ The equation of the tangent line to f at 1 is $y = 2x - 2$. #

7. a. Sketch onto Figure 3 a continuous function that satisfies each of the following properties. (10 points)

- $f(0)=0$
- $f'(x)<0$ over $(-\infty,-3)$ and $(1,5)$
- $f'(x)>0$ over $(-3,1)$ and $(5,\infty)$
- $f''(x)>0$ over $(-1,1)$ and $(1,3)$
- $f''(x)<0$ over $(-\infty,-3)$, $(3,5)$, and $(5,\infty)$!
- $f''(x)=0$ over $(-3,-1)$

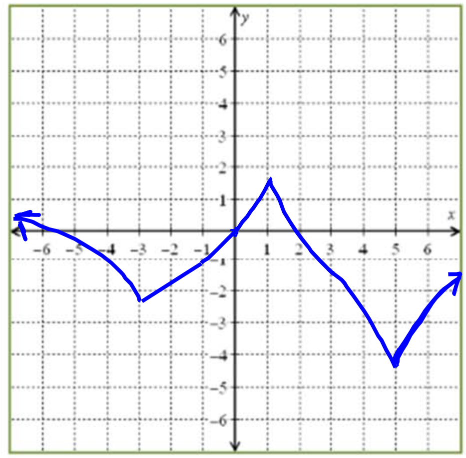
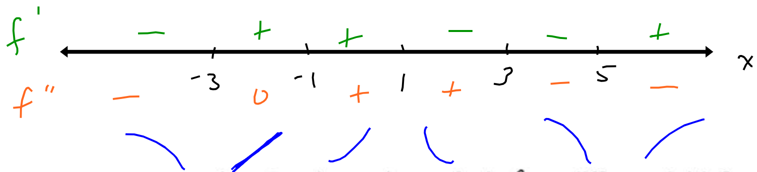


Figure 3: f



b. At what values of x is f nondifferentiable? (1.5 points)

−3, 1, and 5

c. At what values of x are antiderivatives of f nondifferentiable? (1.5 points)

There are none

8. A function f is shown in Figure 4. A certain antiderivative of f, called F, passes through the origin and the absolute maximum value ever achieved by F is 5. Sketch F onto Figure 5. (10 points)

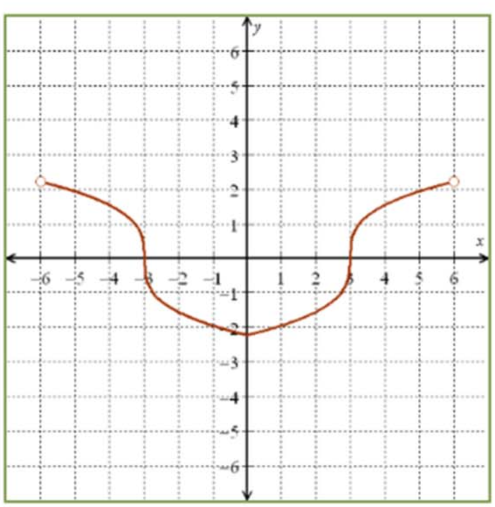


Figure 4: f

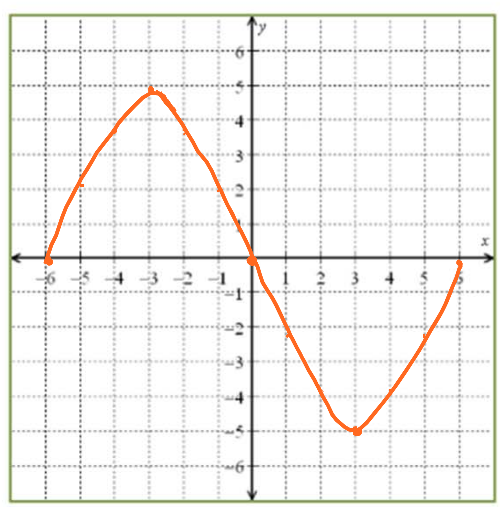


Figure 5: F

9. Answer each question on this page in reference to a function g whose first derivative is shown in Figure 6. You do not need to state how you made your determination; just state the interval(s) or values of x that satisfy the stated property. (2 points each)

Note: The answer to one or more of these questions may be "There is no way of knowing."

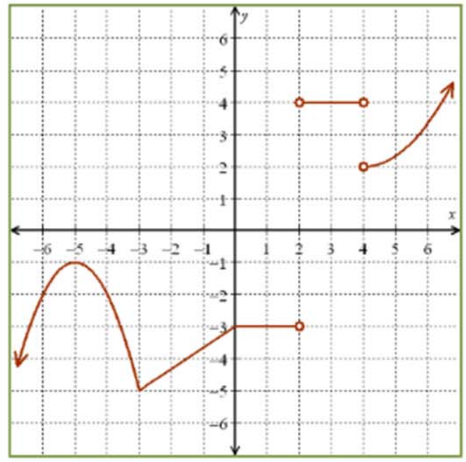


Figure 6: g'

a. Over these intervals, g'' is positive and increasing.

$(4, \infty)$

b. At these values of x, g is nondifferentiable.

2 and 4

c. Over these intervals, g is never negative.

There is no way to tell.

d. At these values of x, g' is nondifferentiable.

-3, 0, 2, and 4

e. Over these intervals, the value of g'' is constant.

$(-3, 0)$, $(0, 2)$, and $(2, 4)$

f. Over these intervals, g is linear.

$(0, 2)$ and $(2, 4)$

g. Over these intervals, antiderivatives of g are linear.

There are no such intervals

h. Over these intervals, g''' is never negative.

$[-3, \infty)$

i. Suppose that $g(3) = 7$. What, then, is the equation of the tangent line to g at the point where $x = 3$?

$y = 4x - 5$