MTH 251 – Test 1
Given October 15, 2015 Name Key

All work on this test will be evaluated for your style of presentation as well as for the "correctness" of your "answer." Follow the writing guidelines established during lecture and lab.

1. Evaluate the following limit showing each step in the process (one step at a time) and stating each limit law used at the time of its use. Specifically, evaluate $\lim\limits_{x\to\infty}\dfrac{x^2-12x+35}{3x^2+10x+7}$. (9 points)

$$\lim_{x\to\infty}\frac{x^2-12x+35}{3x^2+10x+7}=\lim_{x\to\infty}\left(\frac{x^2-12x+35}{3x^2+10x+7}\cdot\frac{1/x^2}{1/x^2}\right)$$

$$=\lim_{x\to\infty}\frac{1-12/x+35/x^2}{3+10/x+7/x^2}$$

$$=\frac{\lim\limits_{x\to\infty}\left(1-12/x+35/x^2\right)}{\lim\limits_{x\to\infty}\left(3+10/x+7/x^2\right)} \qquad \text{LLA5}$$

$$=\frac{\lim\limits_{x\to\infty}1-\lim\limits_{x\to\infty}\frac{12}{x}+\lim\limits_{x\to\infty}\frac{35}{x^2}}{\lim\limits_{x\to\infty}3+\lim\limits_{x\to\infty}\frac{10}{x}+\lim\limits_{x\to\infty}\frac{7}{x^2}} \qquad \text{LLA1} \quad \text{LLA2}$$

$$=\frac{1-0+0}{3+0+0} \qquad \text{LLR2} \quad \text{LLR3}$$

$$=1/3$$

2. Evaluate the following limit showing each step in the process (one step at a time) and stating each limit law used at the time of its use. Specifically, evaluate $\lim_{t \to -1} \frac{5t^2 + 5t}{t^2 - 2t + 3}$. (9 points)

$$\lim_{t \to -1} \frac{5t^2 + 5t}{t^2 - 2t - 3} = \lim_{t \to -1} \frac{5t\,\cancel{(t+1)}}{(t-3)\cancel{(t+1)}}$$

$$= \lim_{t \to -1} \frac{5t}{t-3} \qquad \text{LLA7}$$

$$= \frac{\lim_{t \to -1} (5t)}{\lim_{t \to -1} (t-3)} \qquad \text{LLA5}$$

$$= \frac{5 \lim_{t \to -1} t}{\lim_{t \to -1} t - \lim_{t \to -1} 3} \qquad \text{LLA3} \quad \text{LLA2}$$

$$= \frac{5(-1)}{-1-3} \qquad \text{LLR1} \quad \text{LLR2}$$

$$= 5/4$$

3. Use limit laws to formally establish the value of $\lim_{\theta \to \frac{\pi}{2}} \frac{\sin(2\theta)}{\cos(\theta)}$. **Recall:** $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

(7 points)

$$\lim_{\theta \to \frac{\pi}{2}} \frac{\sin(2\theta)}{\cos(\theta)} = \lim_{\theta \to \frac{\pi}{2}} \frac{2\sin(\theta)\cancel{\cos(\theta)}}{\cancel{\cos(\theta)}}$$

$$= \lim_{\theta \to \frac{\pi}{2}} \left[2\sin(\theta)\right] \qquad LLA7$$

$$= 2\sin\left(\lim_{\theta \to \frac{\pi}{2}} \theta\right) \qquad LLA6$$

$$= 2\sin(\pi/2) \qquad LL\,R1$$

$$= 2 \;\#$$

4. Use limit laws to formally establish the value of $\lim_{x \to 1} \sqrt{\frac{\ln(x^9)}{\ln(x^4)}}$. (6 points)

$$\lim_{x \to 1} \sqrt{\frac{\ln(x^9)}{\ln(x^4)}} = \lim_{x \to 1} \sqrt{\frac{9\cancel{\ln(x)}}{4\cancel{\ln(x)}}}$$

$$= \lim_{x \to 1} \sqrt{\frac{9}{4}} \qquad LLA7$$

$$= \lim_{x \to 1} \frac{3}{2}$$

$$= 3/2 \;\# \qquad LL\,R2$$

5. State the three conditions that must be true about a function, f, when $x = a$, if f is continuous at $x = a$. **Assign numbers to each condition and reference these numbers when answering question 6.** (5 points)

i) f must be defined at a

ii) $\lim_{x \to a} f(x)$ must exist

iii) $\lim_{x \to a} f(x)$ must equal $f(a)$

6. State each value of x where the function g (shown in Figure 1) is discontinuous and in each case state any all conditions of the definition of continuity at a point that fail at that value of x. You may assume that all discontinuities occur at integer values of x. (11 points)

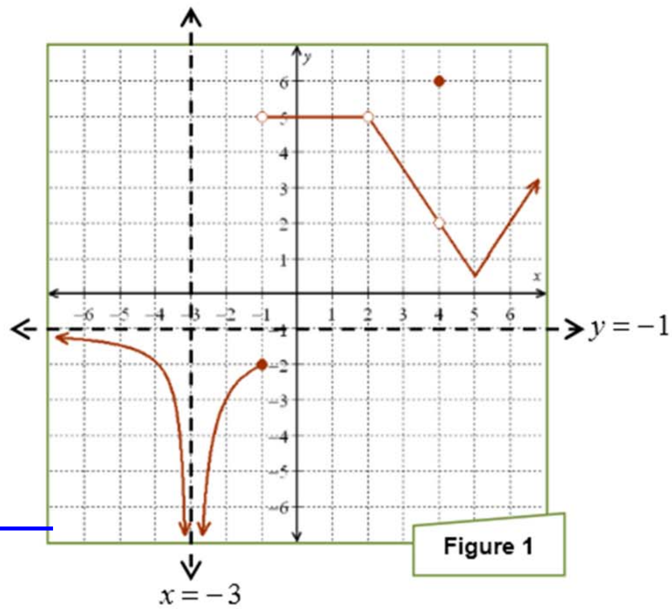

Figure 1

Table 1. Discontinuities on f

Value of x	Failed properties
-3	i, ii, iii
-1	ii, iii
2	i, iii
4	iii

7. Again referring to Figure 1, fill into each blank a number or expression that makes the completed statement true. (7.5 points)

a. The values of x where g is continuous from one side but not the other are <u>-1 only</u>.

b. The values of x where g has removable discontinuities are <u>2 and 4</u>.

c. $\lim_{x \to -1^+} g(x) =$ <u>5</u> d. $\lim_{x \to 2} [g(x) + 4] =$ <u>9</u> e. $\lim_{x \to -3} \frac{g(x)}{g(x)} =$ <u>1</u>

8. Suppose that f is a function whose difference quotient simplifies as $\dfrac{f(x+h)-f(x)}{h}=1.2x-3h$ where $h \neq 0$. Use this information to determine the value of $\dfrac{f(7.5)-f(4)}{3.5}$. Show the relevant work and make sure that your conclusion is clear. (6 points)

$$\frac{f(7.5)-f(4)}{3.5} = \frac{f(4+3.5)-f(4)}{3.5}$$

$$= 1.2(4) - 3(3.5)$$

$$= -5.7$$

9. Consider the function $g(x)=x^2-3x$. Use the definition of the first derivative to find the value of $g'(4)$. You do **not** need to show all of the limit law steps. (12 points)

$$g'(4) = \lim_{h \to 0} \frac{g(4+h)-g(4)}{h}$$

$$= \lim_{h \to 0} \frac{[(4+h)^2 - 3(4+h)] - [4^2 - 3(4)]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{16} + 8h + h^2 \cancel{-12} - 3h - \cancel{4}}{h}$$

$$= \lim_{h \to 0} \frac{5h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(5+h)}{h}$$

$$= \lim_{h \to 0} (5+h)$$

$$= 5$$

10. Draw onto Figure 2 a function, f, that satisfies each and every one of the following properties. Make sure that you draw and appropriately label all asymptotes. (15 points)

- The function f has exactly three discontinuities. (That is, there are discontinuities at exactly three values of x.)
- The only x-intercept on f occurs at -5.
- The y-intercept on f is $(0,3)$.
- $f(2)=2$ and $f(3)=-2$
- $\lim_{x\to-\infty} f(x)=-1$ and $\lim_{x\to\infty} f(x)=\infty$
- $\lim_{x\to-4^-} f(x)=-\infty$ and $\lim_{x\to-4^+} f(x)=\infty$
- $\lim_{x\to-2} f(x)=3$ and $\lim_{x\to3} f(x)=1$
- f is continuous at -2
- f is continuous from the right at 2
- f has constant slope on $[-2,2)$

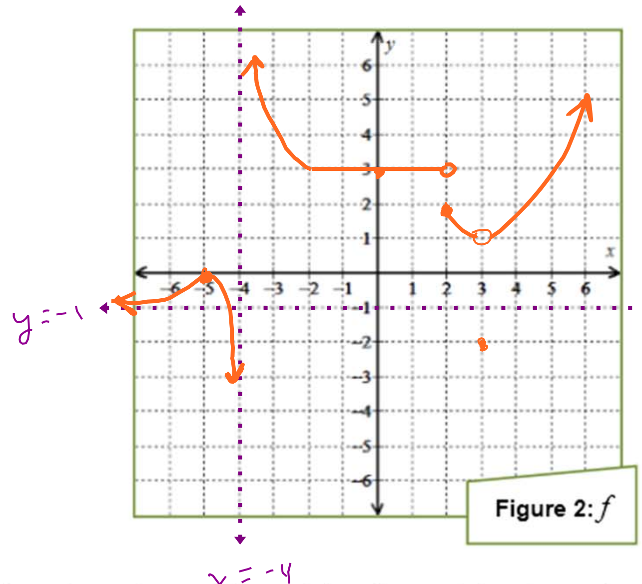

Figure 2: f

11. Suppose that $h(t)=50-t^2$ is a position function where the position unit is miles and the time unit is days. What, including unit, is the average velocity that occurs over the interval $[1,3]$? (6.5 points)

$$\frac{h(3)-h(1)}{3-1} = \frac{41-49}{2}$$

$$= -4$$

The average velocity over [1,3] is -4 mi/day

12. Suppose that v is a velocity function where the velocity unit is ft/s and the time unit is s. What, including unit, would the value of $\frac{v(7)-v(0)}{7}$ tell you? (6.5 points)

The value of $\frac{v(7)-v(0)}{7}$ would tell me the average rate of change in velocity (average acceleration) over the first seven seconds of motion (assuming that motion starts at $t=0$). The unit is $\frac{ft/s}{s}$.