Another example of the classic related rates problem solving strategy
At noon one day a truck is 250 miles due east of a car. The truck is travelling west at a constant speed of 25 mph and the car is travelling due north at a constant speed of 50 mph. At what rate is the distance between the two vehicles changing 15 minutes after noon?

**Agenda 1:** What are our variables? Please note that we are given the value of exactly one rate in this problem, so we need exactly 2 variables; one variable that differentiates to the given rate and one variable that differentiates to the asked for rate. Carefully define each variable (including the time variable). Make sure that you include correct units in your definitions. Angles must always be defined with a unit of radians.

**Agenda 2:** What is our relation equation? A diagram of the problem situation with the variable pieces labeled as variables and constant pieces labeled with their constant values is usually helpful with this agenda.

**Agenda 3:** What is our rate equation? This is derived by differentiating both sides of the relation equation with respect to time.

**Agenda 4:** Account for each variable and each rate in the rate equation. We are interested in what is happening at one specific instant; state the values of each rate and variable at this instant (except, of course, the unknown rate). Make sure that you state negative rates for pieces that are getting smaller over time.

**Agenda 5:** Plug the known values and rates into the rate equation and solve for the unknown rate.

**Agenda 6:** State a contextual conclusion that includes the rate unit. Make sure that your conclusion clearly communicates whether the object is increasing or decreasing.

\[
\text{ Rates: } \\
\frac{dy}{dt} = 50 \\
\frac{dy}{dt} = -25 \\
\text{Unknown Rate: } \frac{dz}{dt} \bigg|_{t=0.25} = 25.5 \\
\]

Let \( x, y, \) and \( t \) be the lengths (mi) and hours after noon.
Relaiton Equation
\[ x^2 + y^2 = z^2 \]

Rate Equation
\[ \frac{d}{dt} (x^2 + y^2) = \frac{d}{dt} (z^2) \]
\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt} \]

At quarter past noon
\[ x = \frac{1}{4} (50) = 12.5 \quad \frac{dx}{dt} = 50 \]
\[ y = 250 - \frac{1}{4} (25) = 243.75 \quad \frac{dy}{dt} = -25 \]
\[ z = \sqrt{12.5^2 + 243.75^2} \quad \frac{dz}{dt} = ? \]
\[ = \sqrt{59570.3125} \]
\[ 2(12.5)(50) + 2(243.75)(-25) = 2\sqrt{59570.3125} \frac{dz}{dt} \]
\[ -22.4 \approx \frac{dz}{dt} \]

At quarter past noon, the distance between the car and the truck was decreasing at about 22.4 mph.
A light house sits on an island of the coast of Maine. The beam source is exactly two miles from the coast along the perpendicular line from the beam source to the coast. The beam makes one complete revolution every 10 seconds. Find the rate at which the beam moves along the coast at the instant the angle between the beam and the perpendicular line to the coast is 50°. To solve this problem we must assume that the coastline is perfectly straight. We must also assume that the light beam is laser like and at ground level.

\[
\text{Rates}
\]
\[
\frac{dx}{dt} = \left(\frac{1 \text{ rev}}{10 \text{ s}}\right) \cdot \left(\frac{\pi \text{ rad}}{1 \text{ rev}}\right) = \frac{\pi}{5}
\]

\[
\text{Find } \frac{dx}{dt} \bigg|_{\alpha = 50^\circ}
\]

Let \( x \) be the length (mi) and \( \alpha \) the angle measured rad. Since Clinton was inaugurated the second time...

**Relation equation**
\[
\tan (\alpha) = \frac{x}{2}
\]

**Rate equation**
\[
\frac{d}{dt} \left[ \tan (\alpha) \right] = \frac{dx}{dt} \cdot \frac{1}{2}
\]
\[
\sec^2 (\alpha) \frac{d\alpha}{dt} = \frac{1}{2} \frac{dx}{dt}
\]

When \( \alpha = 50^\circ \)

\[
\alpha = \frac{5\pi}{18}
\]
\[
\frac{dx}{dt} = \left(\frac{1}{\cos^2 (50^\circ)}\right)^{\frac{1}{2}} \cdot \frac{\pi}{5} = \frac{1}{2} \frac{dx}{dt}
\]

\[
\frac{dx}{dt} = ?
\]
\[
\frac{3.04}{s} \approx \frac{dx}{dt} \text{ At the instant } \alpha = 50^\circ
\]

the light beam is moving along the coast at a clip of about 3.04 mi/s
A tank filled with water is in the shape of an inverted cone 20 feet high with a circular base (on top) whose radius is 5 feet. Water is running out the bottom of the tank at the constant rate of 2 ft³/min. How fast is the water level falling when the water is 8 feet deep?

\[ V = \frac{\pi}{3} r^2 h \]

Let \( V \) (ft³), \( r \) (ft) and \( h \) (ft) be the volume, radius, and height of the water in the cone \( t \) minutes after drainage begins.

\[ \frac{dV}{dt} = -2 \]

Find \( \frac{dh}{dt} \) when \( h = 8 \)

\[ V = \frac{\pi}{3} r^2 h = \frac{\pi}{48} h^3 \]

**Similar triangles!**

\[ \frac{h}{20} = \frac{r}{5} \Rightarrow r = \frac{h}{4} \]

\[ \frac{\partial}{\partial t} \left( \frac{\pi}{3} r^2 h \right) = \frac{\pi}{16} h \frac{d}{dt} \left( \frac{1}{48} h^3 \right) \]

\[ \frac{dV}{dt} = \frac{\pi}{16} h^2 \frac{dh}{dt} \]

When the water level is at 8 ft, it is dropping at the rate of approximately 0.016 ft/min.