

Section V: Quadratic Equations and Functions

Module 4: Applications Involving Quadratic Functions



EXAMPLE: Peter's Plymouth travels 200 miles averaging a certain speed. If the car had gone 10 mph faster, the trip would have taken 1 hour less. Find Peter's average speed.

SOLUTION:

Let r represent Peter's average speed in mph and let t represent the amount of time (in hours) the 200-mile trip took. Since the information given in the problem involves distance, rate, and time, we will use the formula $d = rt$ (*distance = rate \cdot time*). First, let's organize what we know in a table:

distance	rate	time
200	r	t
200	$r + 10$	$t - 1$

(This row represents information about the actual two hundred mile trip.)

(This row represents information about the trip if the car had gone 10 mph faster.)

Using the formula $d = rt$ we obtain the following equations:

$$200 = rt \quad \text{and} \quad 200 = (r + 10)(t - 1).$$

Since we are trying to find Peter's average speed, we need to eliminate the time variable. If we solve $200 = rt$ for t , we obtain the equation

$$t = \frac{200}{r},$$

so we can substitute the expression $\frac{200}{r}$ for t in the equation $200 = (r + 10)(t - 1)$:

$$200 = (r + 10)\left(\frac{200}{r} - 1\right).$$

Now let's solve this equation for r :

$$\begin{aligned} 200 &= (r + 10)\left(\frac{200}{r} - 1\right) \\ \Rightarrow 200 &= 200 - r + \frac{2000}{r} - 10 \\ \Rightarrow 200 &= 190 - r + \frac{2000}{r} \\ \Rightarrow 200 - 190 &= 190 - 190 - r + \frac{2000}{r} \end{aligned}$$

$$\begin{aligned}
&\Rightarrow 10 = -r + \frac{2000}{r} \\
&\Rightarrow 10 \cdot r = \left(-r + \frac{2000}{r}\right) \cdot r \\
&\Rightarrow 10r = -r^2 + 2000 \\
&\Rightarrow r^2 + 10r - 2000 = 0 \\
&\Rightarrow (r + 50)(r - 40) = 0 \\
&\Rightarrow r = -50 \quad \text{or} \quad r = 40
\end{aligned}$$

Since a negative value does not make sense for an average speed, we can say that Peter's average speed was only 40 mph. (There must have been some rough terrain or Peter is just a slow driver!)



EXAMPLE: Assume that the volume of water (in liters) remaining in a full bathtub t minutes after you pull the plug in the drain is represented by the function $V(t) = 2.4t^2 - 24t + 60$.

- a. How much water was in the tub when you pulled the plug?
- b. When will the tub be empty?
- c. In the real world, the number of liters of water in the tub can never be negative. What does the model predict is the least amount of water in the tub? Is this number reasonable?
- d. Draw a graph of the function in the appropriate domain. Use a dotted curve for any portion of the graph that is outside the reasonable domain.
- e. What is the reasonable domain and range for this model?
- f. Why is a quadratic function more reasonable for this problem than a linear function would be?

SOLUTION:

- a. To find the amount of water in the tub when you pulled the plug, we need to find the volume in the tub when $t = 0$ so we just need to compute $V(0)$. Since $V(0) = 60$, we know that there were 60 liters of water in the tub when you pulled the plug.

- b. The tub will be empty when $V(t) = 0$, so we need to solve $2.4t^2 - 24t + 60 = 0$. We can use the quadratic formula to solve this equation:

$$\begin{aligned} t &= \frac{-(-24) \pm \sqrt{(-24)^2 - 4(2.4)(60)}}{2(2.4)} \\ &= \frac{24 \pm \sqrt{0}}{4.8} \\ &= 5 \end{aligned}$$

Thus, the tub will be empty in 5 minutes.

- c. Since the graph of $y = V(t)$ is a parabola that opens upward, we know that the least amount of water that this model predicts will occur at the vertex.

$$t = -\frac{b}{2a} = -\frac{-24}{2(2.4)} = 5 \quad \text{and} \quad V(5) = 2.4(5)^2 - 24(5) + 60 = 0$$

So the vertex is $(5, 0)$, so the model predicts that the least amount of water in the tub is 0 liters, which is reasonable. [Note: If $V(t)$ were negative at the vertex, it would not have been reasonable and we would not include this t -value in the domain of our model.]

- d. The model is only valid on the interval $[0, 5]$, so we will highlight this portion of the curve.

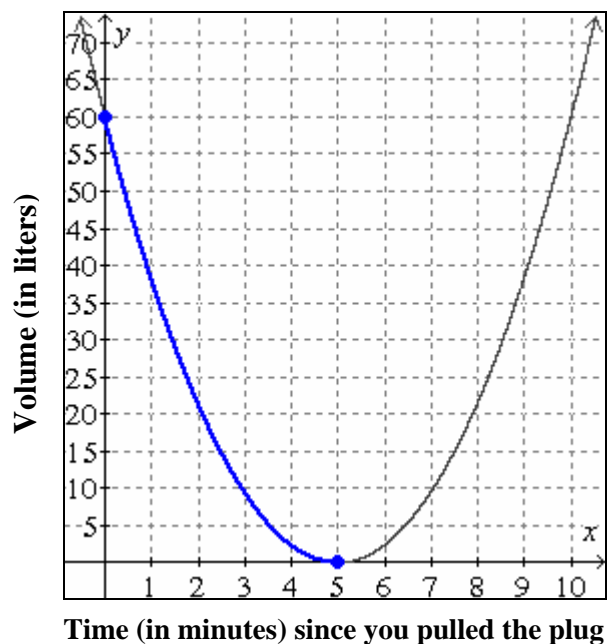


Figure 1: The graph of $y = V(t)$.

- e. As mentioned in part d, the reasonable domain for this model is $[0, 5]$. The reasonable range for this model is $[0, 60]$ which can be seen in Figure 1. We also could have deduced this without the graph because we found in part a that there were 60 liters of water in the tub when you pulled the plug.
 - f. A quadratic function is more reasonable than a linear function since a linear function would imply that the water was draining at a constant rate. Since there is more pressure when the tub is fuller, the water should drain more quickly when you first pull the plug.
-