Section IV: Radical Expressions, Equations, and Functions

Module 6: Solving Radical Equations

EXAMPLE: Solve the equation $\sqrt{x} = 5$ for x.

SOLUTION: To solve this equation, we need to find a number x whose square root is 5. This one we can do in our heads since we know that the square root of 25 is 5. But we won't always be able to solve radical equations in our heads, so let's see how we can solve such this equation using algebra.

Since we need to determine what x needs to be, we need to get x out from underneath the radical sign. Since "squaring" is the opposite of "square rooting," we can square both sides of the equation:

$$\begin{aligned} \sqrt{x} &= 5 \\ \Rightarrow \left(\sqrt{x}\right)^2 &= (5)^2 \qquad \text{(square both sides of the equation)} \\ \Rightarrow x &= 25 \end{aligned}$$

CHECK SOLUTION:

$$\begin{aligned} x = 25: \quad \sqrt{x} &= 5 \\ \sqrt{25} &\overset{?}{=} 5 \\ 5 &= 5 \end{aligned}$$

Since the 25 checks, we can conclude that the solution set is $\{25\}$.

EXAMPLE: Solve the equation $\sqrt[3]{t} = 4$ for t.

SOLUTION: To solve this equation, we need to get t out from under the radical sign. Since the radical sign is a cube root, we will need to cube both sides of the equation.

$$\begin{aligned} \sqrt[3]{t} &= 4 \\ \Rightarrow \left(\sqrt[3]{t}\right)^3 &= (4)^3 \qquad \text{(cube both sides of the equation)} \\ \Rightarrow t &= 64 \end{aligned}$$

CHECK SOLUTION:

$t = 64$:

$$\sqrt[3]{t} = 4$$
$$\sqrt[3]{64} \ ? \ 4$$
$$\sqrt[3]{4^3} \ ? \ 4$$
$$4 = 4$$

Since the 64 checks, we can conclude that the solution set is $\{64\}$.

EXAMPLE: Solve the equation $\sqrt{x} + 3 = 12$ for x.

SOLUTION: To solve this equation we will want to square both sides in order to get x out from under the square root sign. But before we do this we must first **isolate the radical expression** by subtracting 3 from both sides of the equation.

$$\sqrt{x} + 3 = 12$$
$$\Rightarrow \sqrt{x} + 3 - 3 = 12 - 3 \qquad \text{(subtract 3 from both sides of the equation)}$$
$$\Rightarrow \sqrt{x} = 9$$
$$\Rightarrow \left(\sqrt{x}\right)^2 = (9)^2 \qquad \text{(square both sides of the equation)}$$
$$\Rightarrow x = 81$$

CHECK SOLUTION:

$x = 25$:

$$\sqrt{x} + 3 = 12$$
$$\sqrt{81} + 3 \ ? \ 12$$
$$9 + 3 \ ? \ 12$$
$$12 = 12$$

Since the 81 checks, we can conclude that the solution set is $\{81\}$.

EXAMPLE: Solve the equation $\sqrt{2m+5} - 10 = 1$ for m.

SOLUTION: To solve this equation we must first isolate the radical expression, and then square both sides to eliminate the radical.

$$
\begin{aligned}
& \sqrt{2m+5} - 10 = 1 \\
\Rightarrow\ & \sqrt{2m+5} - 10 + 10 = 1 + 10 && \text{(add 10 to both sides)} \\
\Rightarrow\ & \sqrt{2m+5} = 11 \\
\Rightarrow\ & \left(\sqrt{2m+5}\right)^2 = (11)^2 && \text{(square both sides)} \\
\Rightarrow\ & 2m + 5 = 121 \\
\Rightarrow\ & 2m + 5 - 5 = 121 - 5 && \text{(isolate } m\text{)} \\
\Rightarrow\ & 2m = 116 \\
\Rightarrow\ & m = 58
\end{aligned}
$$

CHECK SOLUTION:

$$
\begin{aligned}
m = 58: \quad & \sqrt{2m+5} - 10 = 1 \\
& \sqrt{2(58)+5} - 10 \ ?\ 1 \\
& \sqrt{116+5} - 10 \ ?\ 1 \\
& \sqrt{121} - 10 \ ?\ 1 \\
& 11 - 10 \ ?\ 1 \\
& 1 = 1
\end{aligned}
$$

Since the 58 checks, we can conclude that the solution set is $\{58\}$.

EXAMPLE: Solve the equation $3 = \sqrt{2x + 14} - x$ for x.

SOLUTION: To solve this equation, first isolate the radical expression and then square both sides to eliminate the radical.

$$3 = \sqrt{2x + 14} - x$$

$$\Rightarrow \quad 3 + x = \sqrt{2x + 14} - x + x \qquad \text{(add } x \text{ to both sides)}$$

$$\Rightarrow \quad 3 + x = \sqrt{2x + 14}$$

$$\Rightarrow \quad (3 + x)^2 = \left(\sqrt{2x + 14}\right)^2 \qquad \text{(square both sides)}$$

$$\Rightarrow \quad x^2 + 6x + 9 = 2x + 14$$

$$\Rightarrow \quad x^2 + 6x - 2x + 9 - 14 = 2x - 2x + 14 - 14 \qquad \text{(set equation equal to zero)}$$

$$\Rightarrow \quad x^2 + 4x - 5 = 0$$

$$\Rightarrow \quad (x + 5)(x - 1) = 0$$

$$\Rightarrow \quad x = -5 \quad \text{or} \quad x = 1$$

CHECK SOLUTIONS:

$x = -5$: $3 = \sqrt{2x + 14} - x$

$3 \,?\, \sqrt{2(-5) + 14} - (-5)$

$3 \,?\, \sqrt{-10 + 14} + 5$

$3 \,?\, \sqrt{4} + 5$

$3 \neq 7$

So -5 is not a solution.

$x = 1$: $3 = \sqrt{2x + 14} - x$

$3 \,?\, \sqrt{2(1) + 14} - (1)$

$3 \,?\, \sqrt{2 + 14} - 1$

$3 \,?\, \sqrt{16} - 1$

$3 = 3$

So 1 is a solution.

Since only 1 checks, the solution set is $\{1\}$. This example shows us that even when we do the math correctly, we can find INCORRECT solutions. Hopefully, this will convince you to **always check your solutions**!