Section IV: Radical Expressions, Equations, and Functions

Module 3: Multiplying Radical Expressions

Recall the property of exponents that states that $a^m b^m = (ab)^m$. We can use this rule to obtain an analogous rule for radicals:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = a^{1/n} \cdot b^{1/n}$$

$$= (ab)^{1/n} \qquad \text{(using the property of exponents given above)}$$

$$= \sqrt[n]{ab}$$

Product Rule for Radicals

If a and b are positive real numbers and n is a positive integer, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

EXAMPLE: Perform the indicated multiplication, and simplify completely.

a. $\sqrt{2} \cdot \sqrt{18}$ **b.** $\sqrt[4]{3x^2} \cdot \sqrt[4]{27x^2}$

SOLUTIONS:

a. $\sqrt{2} \cdot \sqrt{18} = \sqrt{2 \cdot 18}$ (product rule for radicals)

$= \sqrt{36}$

$= \sqrt{6^2}$ (write 36 as a perfect square)

$= 6$

b. $\sqrt[4]{3x^2} \cdot \sqrt[4]{27x^2} = \sqrt[4]{3x^2 \cdot 27x^2}$ (product rule for radicals)

$= \sqrt[4]{3 \cdot 27 \cdot x^2 \cdot x^2}$

$= \sqrt[4]{81x^4}$

$= \sqrt[4]{81} \cdot \sqrt[4]{x^4}$ (product rule for radicals)

$= 3|x|$ (we need to use the absolute value since 4 is even)

Product Rule for Simplifying Radical Expressions:

When simplifying a radical expression it is often necessary to use the following equation which is equivalent to the product rule:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \, .$$

EXAMPLE: Simplify $\sqrt{40}$.

SOLUTION: Since 40 isn't a perfect square, we need to write 40 as a product containing a factor that is a perfect square:

$$\begin{aligned} \sqrt{40} &= \sqrt{4 \cdot 10} && \text{(factor 40 using perfect square(s))} \\ &= \sqrt{4} \cdot \sqrt{10} && \text{(product rule for simplifying radical expressions)} \\ &= 2\sqrt{10} \end{aligned}$$

EXAMPLE: Simplify the following.

a. $\sqrt[3]{24}$ **b.** $\sqrt[4]{16w^8}$ **c.** $\sqrt{54d^5}$

SOLUTIONS:

a.
$$\begin{aligned} \sqrt[3]{24} &= \sqrt[3]{8 \cdot 3} && \text{(factor 24 using perfect cube(s))} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{3} && \text{(product rule for simplifying radical expressions)} \\ &= 2\sqrt[3]{3} \end{aligned}$$

b.
$$\begin{aligned} \sqrt[4]{16w^8} &= \sqrt[4]{16} \cdot \sqrt[4]{w^8} && \text{(product rule for simplifying radical expressions)} \\ &= \sqrt[4]{2^4} \cdot \sqrt[4]{\left(w^2\right)^4} \\ &= 2w^2 && \text{(we don't need the absolute value here since } w^2 \text{ must be positive)} \end{aligned}$$

c. $\sqrt{54d^5} = \sqrt{9 \cdot 6 \cdot d^4 \cdot d}$

$= \sqrt{9} \cdot \sqrt{6} \cdot \sqrt{d^4} \cdot \sqrt{d}$ (product rule for simplifying radical expressions)

$= 3d^2\sqrt{6d}$

Try these yourself and check your answers.
Perform the indicated multiplication, and simplify completely.

a. $\sqrt{14} \cdot \sqrt{21}$. **b.** $\sqrt[3]{3y^2} \cdot \sqrt[3]{9y}$.

SOLUTIONS:

a. $\sqrt{14} \cdot \sqrt{21} = \sqrt{14 \cdot 21}$

$= \sqrt{2 \cdot 7 \cdot 3 \cdot 7}$

$= \sqrt{7^2 \cdot 6}$

$= 7\sqrt{6}$

b. $\sqrt[3]{3y^2} \cdot \sqrt[3]{9y} = \sqrt[3]{3y^2 \cdot 9y}$

$= \sqrt[3]{27y^3}$

$= \sqrt[3]{3^3 y^3}$

$= 3y$

EXAMPLE: Perform the following multiplication: $\sqrt[3]{x} \cdot \sqrt[4]{x}$.

SOLUTION:

The **key step** when the indices of the radicals are different is to write the expressions with rational exponents.

$\sqrt[3]{x} \cdot \sqrt[4]{x} = x^{1/3} \cdot x^{1/4}$ (write with rational exponents)

$= x^{1/3 + 1/4}$ (use a propery of exponents)

$= x^{4/12 + 3/12}$ (create a common denominator for the exponent)

$= x^{7/12}$ (use another property of exponents)

$= \sqrt[12]{x^7}$ (write final answer in radical form to agree with original expression)

Try these yourself and check your answers.
Perform the indicated multiplication, and simplify completely.

a. $\sqrt{t} \cdot \sqrt[8]{t^3}$ **b.** $\sqrt[3]{2p^2} \cdot \sqrt{3p}$

SOLUTIONS:

a. $\sqrt{t} \cdot \sqrt[8]{t^3} = t^{1/2} \cdot t^{3/8}$ (write with rational exponents)

$= t^{1/2 + 3/8}$ (use a property of exponents)

$= t^{4/8 + 3/8}$ (create a common denominator for the exponents)

$= t^{7/8}$ (use another property of exponents)

$= \sqrt[8]{t^7}$ (write final answer in radical notation to agree with the original expression)

b. $\sqrt[3]{2p^2} \cdot \sqrt{3p} = \left(2p^2\right)^{1/3} \cdot (3p)^{1/2}$ (write with rational exponents)

$= \left(2p^2\right)^{2/6} \cdot (3p)^{3/6}$ (create a common denominator for the exponents)

$= \left(\left(2p^2\right)^2\right)^{1/6} \left((3p)^3\right)^{1/6}$

$= \left(4p^4 \cdot 27p^3\right)^{1/6}$

$= \left(108p^7\right)^{1/6}$

$= \sqrt[6]{108p^7}$

$= \sqrt[6]{108p^6 p}$

$= p \cdot \sqrt[6]{108p}$