Section II: Functions, Inequalities, and the Absolute Value

Module 2: Compound Inequalities

EXAMPLE: $1 < 2x + 3 \le 7$ is a **compound inequality** since it can be rewritten as two inequalities:

$$1 < 2x + 3 \quad \text{and} \quad 2x + 3 \le 7$$

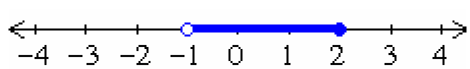
> To solve a compound inequality, rewrite it as two inequalities and solve each of these inequalities. (See the **Section I: Review** if you don't remember how to solve an inequality.)

EXAMPLE: Solve $1 < 2x + 3 \le 7$. Write the solution in interval notation and graph the solution on a number line.

SOLUTION:

$$\begin{array}{lccc} & & 1 < 2x + 3 \le 7 & \\ \Rightarrow & 1 < 2x + 3 & \text{and} & 2x + 3 \le 7 \\ \Rightarrow & 1 - 3 < 2x + 3 - 3 & \text{and} & 2x + 3 - 3 \le 7 - 3 \\ \Rightarrow & -2 < 2x & \text{and} & 2x \le 4 \\ \Rightarrow & \dfrac{-2}{2} < \dfrac{2x}{2} & \text{and} & \dfrac{2x}{2} \le \dfrac{4}{2} \\ \Rightarrow & -1 < x & \text{and} & x \le 2 \end{array}$$

We can combine $-1 < x$ and $x \le 2$ into the compound inequality $-1 < x \le 2$. So, the solution to $1 < 2x + 3 \le 7$ is the set $\{x \mid x \in \mathbb{R} \text{ and } -1 < x \le 2\}$, which we can write in interval notation: $(-1, 2]$. Below is a graph of the solution on a number line:



> Remember that on a graph, an empty circle indicates that the point is not included and a solid circle indicates that the point is included. (So in the graph above, –1 is not included while 2 is included.)

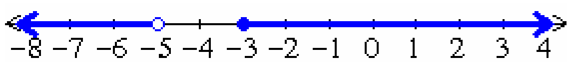
EXAMPLE: Solve the compound inequality $x + 3 < -2$ or $-2x \le 6$. Write the solution in interval notation and graph the solution on a number line.

SOLUTION: Here, the compound inequality is already written in two parts. We'll solve each part independently:

$$
\begin{array}{llcl}
 & x + 3 < -2 & \text{or} & -2x \le 6 \\
\Rightarrow & x + 3 - 3 < -2 - 3 & \text{or} & \dfrac{-2x}{-2} \ge \dfrac{6}{-2} \\
\Rightarrow & x < -5 & \text{or} & x \ge -3
\end{array}
$$

(Multiplying/dividing by a negative number reverses the inequality.)

Thus, the solution to $x + 3 < -2$ or $-2x \le 6$ is $(-\infty, -5)$ or $[-3, \infty)$; a graph of the solution is given below:



Try these yourself and check your answers.

a. Solve $-5 \le -2x + 3 < 7$. Write your solution in interval notation.

b. Solve $3a - 5 < 10$ or $3a - 5 \ge 16$. Write your solution in interval notation.

SOLUTIONS:

a.

$$
\begin{array}{llcl}
 & & -5 \le -2x + 3 < 7 & \\
\Rightarrow & -5 \le -2x + 3 & \text{and} & -2x + 3 < 7 \\
\Rightarrow & -5 - 3 \le -2x + 3 - 3 & \text{and} & -2x + 3 - 3 < 7 - 3 \\
\Rightarrow & -8 \le -2x & \text{and} & -2x < 4 \\
\Rightarrow & \dfrac{-8}{-2} \ge \dfrac{-2x}{-2} & \text{and} & \dfrac{-2x}{-2} > \dfrac{4}{-2} \\
\Rightarrow & 4 \ge x & \text{and} & x > -2 \\
\Rightarrow & & -2 < x \le 4 &
\end{array}
$$

So the solution to $-5 \le -2x + 3 < 7$ is $(-2, 4]$.

b.

$$
\begin{aligned}
& 3a - 5 < 10 && \text{or} && 3a - 5 \ge 16 \\
\Rightarrow\ & 3a - 5 + 5 < 10 + 5 && \text{or} && 3a - 5 + 5 \ge 16 + 5 \\
\Rightarrow\ & 3a < 15 && \text{or} && 3a \ge 21 \\
\Rightarrow\ & a < 5 && \text{or} && a \ge 7
\end{aligned}
$$

The solution to $3a - 5 < 10$ or $3a - 5 \ge 16$ is $(-\infty, 5)$ or $[7, \infty)$.