Section III: Rational Expressions, Equations, and Functions

Module 7: Formulas Involving Rational Expressions

There are many mathematical formulas that are useful in a great variety of applications (e.g., physics, economics, architecture, athletics). As you should have seen in previous mathematics courses, it is important to have the mathematical skills necessary to *solve formulas for specified variables*. Now that we are able to solve rational equations, we can work with formulas that involve rational expressions.

EXAMPLE: The formula $\frac{t}{m}+\frac{t}{n}=1$ represents the total time t required for two people to complete a job together if the times required for the people to complete the job alone are m and n, respectively. (Notice that this is a formula that we utilized in Module 6.) Solve this formula for m.

SOLUTION:

$$\frac{t}{m}+\frac{t}{n}=1$$

$$\Rightarrow \quad m\cdot n\cdot\left(\frac{t}{m}+\frac{t}{n}\right)=1\cdot m\cdot n \qquad \text{(multiply both sides by the LCD)}$$

$$\Rightarrow \quad nt+mt=mn$$

$$\Rightarrow \quad nt=mn-mt \qquad \text{(get all of the } m\text{'s on the same side)}$$

$$\Rightarrow \quad nt=m(n-t) \qquad \text{(factor out } m \text{ on the left-hand side)}$$

$$\Rightarrow \quad \frac{nt}{n-t}=\frac{m\cancel{(n-t)}}{\cancel{n-t}} \qquad \text{(isolate } m\text{)}$$

$$\Rightarrow \quad m=\frac{nt}{n-t}$$

EXAMPLE: The formula $\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$ gives the combined resistance R of an electrical circuit with three resistors (with resistance r_1, r_2, and r_3, respectively) connected in parallel. Solve this formula for r_2.

SOLUTION:

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\Rightarrow \quad r_1 \cdot r_2 \cdot r_3 \cdot R \cdot \frac{1}{R} = \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \right) \cdot r_1 \cdot r_2 \cdot r_3 \cdot R \qquad \text{(multiply both sides by LCD)}$$

$$\Rightarrow \quad r_1 r_2 r_3 = r_2 r_3 R + r_1 r_3 R + r_1 r_2 R$$

$$\Rightarrow \quad r_1 r_2 r_3 - r_2 r_3 R - r_1 r_2 R = r_1 r_3 R \qquad \text{(get all of the } r_2\text{'s on the same side)}$$

$$\Rightarrow \quad r_2 (r_1 r_3 - r_3 R - r_1 R) = r_1 r_3 R \qquad \text{(factor out } r_2 \text{ from the right-hand side)}$$

$$\Rightarrow \quad \frac{r_2 \cancel{(r_1 r_3 - r_3 R - r_1 R)}}{\cancel{r_1 r_3 - r_3 R - r_1 R}} = \frac{r_1 r_3 R}{r_1 r_3 - r_3 R - r_1 R} \qquad \text{(isolate } r_2\text{)}$$

$$\Rightarrow \quad r_2 = \frac{r_1 r_3 R}{r_1 r_3 - r_3 R - r_1 R}$$

Try this one yourself and check your answer.

The formula $a = \dfrac{v_2 - v_1}{t_2 - t_1}$ gives the average acceleration, a, of a moving object when its velocity changes from v_1 at time t_1 to v_2 at time t_2. Solve this formula for t_2.

SOLUTION:

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\Rightarrow \quad (t_2 - t_1) \cdot a = \frac{v_2 - v_1}{t_2 - t_1} \cdot (t_2 - t_1) \qquad \text{(multiply both sides by LCD)}$$

$$\Rightarrow \quad t_2 a - t_1 a = v_2 - v_1$$

$$\Rightarrow \quad t_2 a = v_2 - v_1 + t_1 a$$

$$\Rightarrow \quad \frac{t_2 \cancel{a}}{\cancel{a}} = \frac{v_2 - v_1 + t_1 a}{a} \qquad \text{(isolate } t_2\text{)}$$

$$\Rightarrow \quad t_2 = \frac{v_2 - v_1 + t_1 a}{a}$$

EXAMPLE: Solve the equation $y = \dfrac{4}{x - 3}$ for x.

SOLUTION:

$$y = \frac{4}{x - 3}$$

$$\Rightarrow \quad (x - 3) \cdot y = \frac{4}{\cancel{x - 3}} \cdot \cancel{(x - 3)} \qquad \text{(multiply both sides by LCD)}$$

$$\Rightarrow \quad (x - 3) \cdot y = 4$$

$$\Rightarrow \quad \frac{(x - 3) \cdot \cancel{y}}{\cancel{y}} = \frac{4}{y}$$

$$\Rightarrow \quad x - 3 = \frac{4}{y}$$

$$\Rightarrow \quad x = \frac{4}{y} + 3$$

Try this one yourself and check your answer.

Solve the equation $y = \dfrac{x}{2 + x}$ for x.

SOLUTION:

$$y = \frac{x}{2 + x}$$

$$\Rightarrow (2 + x) \cdot y = \frac{x}{2 + x} \cdot (2 + x) \qquad \text{(multiply both sides by LCD)}$$

$$\Rightarrow \quad 2y + xy = x$$

$$\Rightarrow \quad 2y = x - xy \qquad \text{(get all of the } x\text{'s on the same side)}$$

$$\Rightarrow \quad 2y = x(1 - y) \qquad \text{(factor out } x\text{)}$$

$$\Rightarrow \quad \frac{2y}{1 - y} = \frac{x\,\cancel{(1 - y)}}{\cancel{1 - y}} \qquad \text{(isolate } x\text{)}$$

$$\Rightarrow \quad x = \frac{2y}{1 - y}$$