Section III: Rational Expressions, Equations, and Functions

Module 3: Adding and Subtracting Rational Expressions

Adding and subtracting rational expressions works the same way as adding and subtracting fractions.

EXAMPLE:

ADDING FRACTIONS: SUBTRACTING FRACTIONS:

(create common denominators)

$$\frac{5}{6}+\frac{2}{9}=\frac{5}{6}\cdot\frac{3}{3}+\frac{2}{9}\cdot\frac{2}{2} \qquad\qquad \frac{4}{15}-\frac{11}{6}=\frac{4}{15}\cdot\frac{2}{2}-\frac{11}{6}\cdot\frac{5}{5}$$

$$=\frac{15}{18}+\frac{4}{18} \qquad\qquad =\frac{8}{30}-\frac{55}{30}$$

(add/subtract numerators)

$$=\frac{15+4}{18} \qquad\qquad =\frac{8-55}{30}$$

$$=\frac{19}{18} \qquad\qquad =-\frac{47}{30}$$

The key step in adding or subtracting fractions involves creating **common denominators.** This is also the key step in adding or subtracting rational expressions.

To **add or subtract rational expressions with like denominators**, add or subtract the numerators and keep the same denominator:

$$\frac{A}{C}+\frac{B}{C}=\frac{A+B}{C} \qquad \text{AND} \qquad \frac{A}{C}-\frac{B}{C}=\frac{A-B}{C}$$

It is often useful to factor denominators so that it is easier to create common denominators.

EXAMPLE: Perform the indicated operations; simplify your result completely.

a. $\dfrac{x}{x^2+9x+20}-\dfrac{4}{x^2+7x+12}$

b. $\dfrac{2}{p^2-9}+\dfrac{4}{p+3}$

c. $\dfrac{1}{t+1}-\dfrac{t}{t-2}+\dfrac{t^2+2}{t^2-t-2}$

d. $\dfrac{3y^2}{y-2}-\dfrac{12-12y}{2-y}$

SOLUTIONS:

a. $$\frac{x}{x^2+9x+20}-\frac{4}{x^2+7x+12}=\frac{x}{(x+5)(x+4)}-\frac{4}{(x+4)(x+3)}$$ (factor denominators)

$$=\frac{x}{(x+5)(x+4)}\cdot\frac{x+3}{x+3}-\frac{4}{(x+4)(x+3)}\cdot\frac{x+5}{x+5}$$

$$=\frac{x(x+3)}{(x+5)(x+4)(x+3)}-\frac{4(x+5)}{(x+4)(x+3)(x+5)}$$

$$=\frac{x(x+3)-4(x+5)}{(x+5)(x+4)(x+3)}$$ (subtract numerators)

$$=\frac{x^2-x-20}{(x+5)(x+4)(x+3)}$$

$$=\frac{(x-5)\cancel{(x+4)}}{(x+5)\cancel{(x+4)}(x+3)}$$

$$=\frac{x-5}{(x+5)(x+3)}$$

b. $$\frac{2}{p^2-9}+\frac{4}{p+3}=\frac{2}{(p-3)(p+3)}+\frac{4}{p+3}$$ (factor denominators)

$$=\frac{2}{(p-3)(p+3)}+\frac{4}{p+3}\cdot\frac{p-3}{p-3}$$ (create common denominators)

$$=\frac{2+4\cdot(p-3)}{(p-3)(p+3)}$$ (add numerators)

$$=\frac{2+4p-12}{(p-3)(p+3)}$$

$$=\frac{4p-10}{(p-3)(p+3)}$$

$$=\frac{2(2p-5)}{(p-3)(p+3)}$$

c. $$\frac{1}{t+1}-\frac{t}{t-2}+\frac{t^2+2}{t^2-t-2}=\frac{1}{t+1}-\frac{t}{t-2}+\frac{t^2+2}{(t+1)(t-2)}$$ (factor denominators)

$$=\frac{1}{t+1}\cdot\frac{t-2}{t-2}-\frac{t}{t-2}\cdot\frac{t+1}{t+1}+\frac{t^2+2}{(t+1)(t-2)}$$

$$=\frac{t-2-t(t+1)+t^2+2}{(t+1)(t-2)}$$ (add / subtract numerators)

$$=\frac{t-2-t^2-t+t^2+2}{(t+1)(t-2)}$$

$$=\frac{0}{(t+1)(t-2)}$$

$$=0$$

d. For this example it is helpful to recognize the following fact: $-(a-b) = b-a$

$$\frac{3y^2}{y-2} - \frac{12-12y}{2-y} = \frac{3y^2}{y-2} - \left(\frac{-1}{-1}\right)\cdot\frac{12-12y}{2-y}$$

$$= \frac{3y^2}{y-2} - \frac{-1\cdot(12-12y)}{y-2}$$

$$= \frac{3y^2-(-1)(12-12y)}{y-2}$$

$$= \frac{3y^2+12-12y}{y-2}$$

$$= \frac{3(y^2-4y+4)}{y-2}$$

$$= \frac{3\cancel{(y-2)}(y-2)}{\cancel{y-2}}$$

$$= 3(y-2)$$

Try these yourself and check your answers.
Perform the indicated operations; simplify your result completely.

a. $\dfrac{1}{2x-3} + \dfrac{x}{x-4}$ **b.** $\dfrac{4}{b+3} - \dfrac{b-2}{b^2+2b-3}$

SOLUTIONS:

a. $$\frac{1}{2x-3} + \frac{x}{x-4} = \frac{1}{2x-3}\cdot\frac{x-4}{x-4} + \frac{x}{x-4}\cdot\frac{2x-3}{2x-3}$$ (create common denominators)

$$= \frac{x-4+x(2x-3)}{(2x-3)(x-4)}$$

$$= \frac{x-4+2x^2-3x}{(2x-3)(x-4)}$$

$$= \frac{2x^2-2x-4}{(2x-3)(x-4)}$$

$$= \frac{2(x^2-x-2)}{(2x-3)(x-4)}$$

$$= \frac{2(x+1)(x-2)}{(2x-3)(x-4)}$$

b. $$\frac{4}{b+3} - \frac{b-2}{b^2+2b-3} = \frac{4}{b+3} - \frac{b-2}{(b+3)(b-1)}$$ (factor denominators)

$$= \frac{4}{b+3}\cdot\frac{b-1}{b-1} - \frac{b-2}{(b+3)(b-1)}$$ (create common denominators)

$$= \frac{4(b-1)-(b-2)}{(b+3)(b-1)}$$

$$= \frac{4b-4-b+2}{(b+3)(b-1)}$$

$$= \frac{3b-2}{(b+3)(b-1)}$$

EXAMPLE: If $f(x) = -3x+5$, find and simplify the expression $\dfrac{f(x+h)-f(x)}{h}$.

SOLUTION:

$$\frac{f(x+h)-f(x)}{h} = \frac{-3(x+h)+5-(-3x+5)}{h}$$

$$= \frac{-3x-3h+5+3x-5)}{h}$$

$$= \frac{-3\cancel{h}}{\cancel{h}}$$

$$= -3$$

Try this one yourself and check your answer.

If $g(x) = 7x-10$, find and simplify the expression $\dfrac{g(x+h)-g(x)}{h}$.

SOLUTION:

$$\frac{g(x+h)-g(x)}{h} = \frac{7(x+h)-10-(7x-10)}{h}$$

$$= \frac{7x+7h-10-7x+10}{h}$$

$$= \frac{7\cancel{h}}{\cancel{h}}$$

$$= 7$$

EXAMPLE If $g(x)=\dfrac{4}{3x}$, find and simplify the expression $g(a+2)-g(a)$.

SOLUTION:

$$g(a+2)-g(a)=\frac{4}{3(a+2)}-\frac{4}{3a}$$

$$=\frac{4}{3(a+2)}\cdot\frac{a}{a}-\frac{4}{3a}\cdot\frac{a+2}{a+2} \qquad \text{(create common denominators)}$$

$$=\frac{4a}{3a(a+2)}-\frac{4(a+2)}{3a(a+2)}$$

$$=\frac{4a-4(a+2)}{3a(a+2)}$$

$$=\frac{-8}{3a(a+2)}$$

$$=\frac{-8}{3a(a+2)}$$

Try this one yourself and check your answer.

If $f(x)=\dfrac{1}{2x}$, find and simplify the expression $f(w-1)-f(w+1)$.

SOLUTION:

$$f(w-1)-f(w+1)=\frac{1}{2(w-1)}-\frac{1}{2(w+1)}$$

$$=\frac{1}{2(w-1)}\cdot\frac{w+1}{w+1}-\frac{1}{2(w+1)}\cdot\frac{w-1}{w-1} \qquad \text{(obtain a common denominator)}$$

$$=\frac{w+1-(w-1)}{2(w-1)(w+1)}$$

$$=\frac{w+1-w+1}{2(w-1)(w+1)}$$

$$=\frac{2}{2(w-1)(w+1)}$$

$$=\frac{1}{(w-1)(w+1)}$$