Section III: Rational Expressions, Equations, and Functions

Module 2: Multiplying and Dividing Rational Expressions

MULTIPLYING RATIONAL EXPRESSIONS

Multiplying rational expressions works the same way as multiplying fractions.

 **EXAMPLE:** Multiplying Fractions

$$\frac{16}{45}\cdot\frac{15}{8}=\frac{16\cdot 15}{45\cdot 8}$$ (multiply the numerators and the denominators)

$$=\frac{2\cdot \cancel{8}\cdot \cancel{15}}{3\cdot \cancel{15}\cdot \cancel{8}}$$ (cancel factors that appear in both the numerator and the denominator)

$$=\frac{2}{3}$$

To **multiply two rational expressions**, multiply numerators and multiply denominators:

$$\frac{A}{B}\cdot\frac{C}{D}=\frac{A\cdot C}{B\cdot D}$$

It is often useful to factor the components of the rational expression so that we can cancel all common factors in the numerator and denominator.

 **EXAMPLE:** Perform the indicated multiplication; simplify your result completely.

a. $\dfrac{25a}{9b^8}\cdot\dfrac{6b^5}{5a^2}$

b. $\dfrac{x-1}{x+3}\cdot\dfrac{3x+9}{4x^2-4}$

SOLUTIONS:

a. $\dfrac{25a}{9b^8}\cdot\dfrac{6b^5}{5a^2}=\dfrac{150ab^5}{45b^8a^2}$ (multiply the numerators and the denominators)

$=\dfrac{10\cdot 15\,a\,b^5}{3\cdot 15\,b^5\,b^3\,a\,a}$ (cancel factors that appear in both the numerator and the denominator)

$=\dfrac{10}{3ab^3}$

b. $\dfrac{x-1}{x+3}\cdot\dfrac{3x+9}{4x^2-4}=\dfrac{(x-1)\cdot 3(x+3)}{(x+3)\cdot 4(x^2-1)}$ (multiply the numerators and the denominators)

$=\dfrac{3\,(x-1)\,(x+3)}{4\,(x+3)\,(x-1)\,(x+1)}$ (cancel factors that appear in both the numerator and the denominator)

$=\dfrac{3}{4(x+1)}$

Try these yourself and then check your answers.

Perform the indicated multiplication; simplify your result completely.

a. $\dfrac{16m^2}{12n^8}\cdot\dfrac{9n^{10}}{4m^5}$

b. $\dfrac{p^2-10p+9}{p^2-4}\cdot\dfrac{3p+6}{p-9}$

SOLUTIONS:

a. $\dfrac{16m^2}{12n^8}\cdot\dfrac{9n^{10}}{4m^5}=\dfrac{16\cdot 9m^2n^{10}}{12\cdot 4n^8m^5}$ (multiply the numerators and the denominators)

$=\dfrac{4\cdot 4\cdot 3\cdot 3\,m^2\,n^8\,n^2}{3\cdot 4\cdot 4\,n^8\,m^2\,m^3}$ (cancel factors that appear in both the numerator and the denominator)

$=\dfrac{3n^2}{m^3}$

b. $\dfrac{p^2-10p+9}{p^2-4}\cdot\dfrac{3p+6}{p-9}=\dfrac{(p-1)(p-9)}{(p+2)(p-2)}\cdot\dfrac{3(p+2)}{p-9}$ (factor numerators and denominators)

$=\dfrac{(p-1)\cancel{(p-9)}\cdot 3\cancel{(p+2)}}{\cancel{(p+2)}(p-2)\cdot\cancel{(p-9)}}$ (cancel factors that appear in both the numerator and the denominator)

$=\dfrac{3(p-1)}{p-2}$

DIVIDING RATIONAL EXPRESSIONS

Dividing rational expressions works the same way as dividing fractions.

EXAMPLE: Dividing Fractions

$\dfrac{10}{9}\div\dfrac{20}{27}=\dfrac{10}{9}\cdot\dfrac{27}{20}$ (multiply by the reciprocal of the divisor)

$=\dfrac{10\cdot 27}{9\cdot 20}$

$=\dfrac{\cancel{10}\cdot\cancel{9}\cdot 3}{\cancel{9}\cdot\cancel{10}\cdot 2}$ (factor to facilitate canceling)

$=\dfrac{3}{2}$

To **divide two rational expressions**, multiply by the reciprocal of the divisor (i.e., change division into multiplication by "flipping" the second fraction):

$$\frac{A}{B}\div\frac{C}{D}=\frac{A}{B}\cdot\frac{D}{C}$$

EXAMPLE: Perform the indicated division; simplify your result completely.

a. $\dfrac{p^4}{p-6}\div\dfrac{p^3}{p^2-6p}$

b. $\dfrac{8x^2}{x^2-9}\div\dfrac{x^2-x}{4x+12}$

SOLUTIONS:

a. $\dfrac{p^4}{p-6} \div \dfrac{p^3}{p^2-6p} = \dfrac{p^4}{p-6} \cdot \dfrac{p(p-6)}{p^3}$ (multiply by the reciprocal of the divisor)

$= \dfrac{p^4 \cdot p\,(p-6)}{(p-6) \cdot p^3}$ (cancel factors that appear in both the numerator and the denominator)

$= \dfrac{p^5}{p^3}$

$= p^{5-3}$ (use rule of exponents)

$= p^2$

b. $\dfrac{8x^2}{x^2-9} \div \dfrac{x^2-x}{4x+12} = \dfrac{8x^2}{x^2-9} \cdot \dfrac{4x+12}{x^2-x}$ (multiply by the reciprocal of the divisor)

$= \dfrac{8x \cdot x \cdot 4\,(x+3)}{(x-3)\,(x+3) \cdot x\,(x-1)}$ (cancel factors that appear in both the numerator and the denominator)

$= \dfrac{32x}{(x-3)(x-1)}$

Try these yourself and then check your answers.

Perform the indicated division; simplify your result completely.

a. $\dfrac{6w^2}{2w+10} \div \dfrac{9w^{10}}{w+5}$

b. $\dfrac{t^2-9}{4t-4} \div \dfrac{t^2+t-6}{2t-2}$

SOLUTIONS:

a. $\dfrac{6w^2}{2w+10} \div \dfrac{9w^{10}}{w+5} = \dfrac{6w^2}{2w+10} \cdot \dfrac{w+5}{9w^{10}}$ (multiply by the reciprcal of the divisor)

$= \dfrac{2 \cdot 3\,w^2 \cdot (w+5)}{2\,(w+5) \cdot 3 \cdot 3\,w^2\,w^8}$ (cancel factors that appear in both the numerator and the denominator)

$= \dfrac{1}{3w^8}$

b. $\dfrac{t^2-9}{4t-4} \div \dfrac{t^2+t-6}{2t-2} = \dfrac{t^2-9}{4t-4} \cdot \dfrac{2t-2}{t^2+t-6}$ (multiply by the reciprcal of the divisor)

$= \dfrac{(t+3)(t-3)}{2 \cdot 2(t-1)} \cdot \dfrac{2(t-1)}{(t+3)(t-2)}$ (factor expressions)

$= \dfrac{(t+3)(t-3) \cdot 2(t-1)}{2 \cdot 2(t-1) \cdot (t+3)(t-2)}$ (cancel factors that appear in numerator and denominator)

$= \dfrac{t-3}{2(t-2)}$