SOLUTIONS: Worksheet 5

1. Prove that the equation $\sec(t) - \cos(t) = \tan(t)\sin(t)$ is an identity.
Be sure to organize your proof as shown in the Online Lecture Notes.

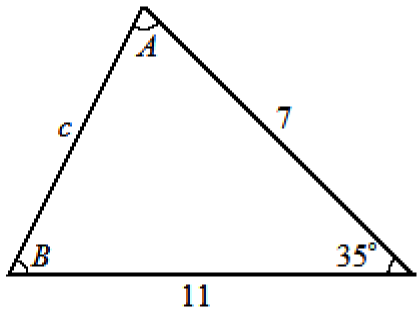
$$
\begin{aligned}
\sec(t) - \cos(t) &= \frac{1}{\cos(t)} - \cos(t)\cdot\frac{\cos(t)}{\cos(t)} \\
&= \frac{1}{\cos(t)} - \frac{\cos^2(t)}{\cos(t)} \\
&= \frac{1-\cos^2(t)}{\cos(t)} \\
&= \frac{\sin^2(t)}{\cos(t)} \qquad \text{(since } 1-\cos^2(t) = \sin^2(t)\text{)} \\
&= \frac{\sin(t)}{\cos(t)}\cdot\frac{\sin(t)}{1} \\
&= \tan(t)\sin(t)
\end{aligned}
$$

2. Prove that the equation $\dfrac{1}{1-\sin(t)}+\dfrac{1}{1+\sin(t)}=2\sec^2(t)$ is an identity.

Be sure to organize your proof as shown in the Online Lecture Notes.

$$\begin{aligned}
\frac{1}{1-\sin(t)}+\frac{1}{1+\sin(t)} &= \frac{1}{1-\sin(t)}\cdot\frac{1+\sin(t)}{1+\sin(t)}+\frac{1}{1+\sin(t)}\cdot\frac{1-\sin(t)}{1-\sin(t)}\\
&= \frac{1+\sin(t)}{\big(1-\sin(t)\big)\big(1+\sin(t)\big)}+\frac{1-\sin(t)}{\big(1+\sin(t)\big)\big(1-\sin(t)\big)}\\
&= \frac{1+\sin(t)+1-\sin(t)}{\big(1-\sin(t)\big)\big(1+\sin(t)\big)}\\
&= \frac{2}{1-\sin^2(t)}\\
&= \frac{2}{\cos^2(t)} \qquad \text{(since } 1-\sin^2(t)=\cos^2(t)\text{)}\\
&= 2\cdot\frac{1}{\cos^2(t)}\\
&= 2\sec^2(t)
\end{aligned}$$

3. Find the missing side c and missing angles A and B for the triangle below (not drawn to scale) Approximations are acceptable here, although they must be denoted correctly.



We can use the law of cosines to find side c.

$$\begin{aligned} c^2 &= 11^2 + 7^2 - 2\cdot 11\cdot 7\cdot\cos(35^\circ) \\ &= 121 + 49 - 154\cos(35^\circ) \\ &= 170 - 154\cos(35^\circ) \end{aligned}$$

Thus,

$$\begin{aligned} c &= \sqrt{170 - 154\cos(35^\circ)} \\ &\approx 6.62 \end{aligned}$$

Now we can use this approximation for c to find (an approximation for) angle B (the smaller of the unknown angles since it's opposite the smaller side) using the law of sines:

$$\begin{aligned} &\frac{\sin(B)}{7} = \frac{\sin(35^\circ)}{c} \\ \Rightarrow\ &\sin(B) = 7\cdot\frac{\sin(35^\circ)}{c} \\ \Rightarrow\ &B = \sin^{-1}\left(7\cdot\frac{\sin(35^\circ)}{c}\right) \\ \Rightarrow\ &B \approx \sin^{-1}\left(7\cdot\frac{\sin(35^\circ)}{6.62}\right) \\ \Rightarrow\ &B \approx 37.3^\circ \end{aligned}$$

Finally, we can find (an approximation for) angle A:

$$\begin{aligned} A &= 180^\circ - 35^\circ - B \\ &\approx 180^\circ - 35^\circ - 37.3^\circ \\ &\approx 107.7^\circ \end{aligned}$$