SOLUTIONS: Worksheet 4

1. Find *all* of the solutions to the equation $4\sin(3\theta)+2=0$. Provide *exact* solutions.

$$4\sin(3\theta)+2 = 0$$

$$\Rightarrow \quad \sin(3\theta) = -\tfrac{2}{4}$$

$$\Rightarrow \quad 3\theta = \sin^{-1}\left(-\tfrac{1}{2}\right)+2k\pi,\ \ k\in\mathbb{Z} \quad \text{or} \quad 3\theta = \pi-\sin^{-1}\left(-\tfrac{1}{2}\right)+2k\pi,\ \ k\in\mathbb{Z}$$

$$\Rightarrow \quad 3\theta = -\tfrac{\pi}{6}+2k\pi,\ \ k\in\mathbb{Z} \quad \text{or} \quad 3\theta = \pi-\left(-\tfrac{\pi}{6}\right)+2k\pi,\ \ k\in\mathbb{Z}$$

$$\Rightarrow \quad \frac{3\theta}{3} = \frac{-\frac{\pi}{6}}{3}+\frac{2k\pi}{3},\ \ k\in\mathbb{Z} \quad \text{or} \quad \frac{3\theta}{3} = \frac{\frac{7\pi}{6}}{3}+\frac{2k\pi}{3},\ \ k\in\mathbb{Z}$$

$$\Rightarrow \quad \theta = -\tfrac{\pi}{18}+\tfrac{2k\pi}{3},\ \ k\in\mathbb{Z} \quad \text{or} \quad \theta = \tfrac{7\pi}{18}+\tfrac{2k\pi}{3},\ \ k\in\mathbb{Z}$$

2. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. Evaluate $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$.

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4} \qquad \left(\text{since } \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \text{ and } -\frac{\pi}{2} \le -\frac{\pi}{4} \le \frac{\pi}{2}\right)$$

b. Evaluate $\cos\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$.

$$\begin{aligned} \cos\left(\cos^{-1}\left(-\tfrac{1}{2}\right)\right) &= \cos\left(\tfrac{2\pi}{3}\right) && \left(\text{since } \cos\left(\tfrac{2\pi}{3}\right) = -\tfrac{1}{2} \text{ and } 0 \le \tfrac{2\pi}{3} \le \pi\right) \\ &= -\tfrac{1}{2} && \left(\text{since } \cos\left(\tfrac{2\pi}{3}\right) = -\tfrac{1}{2}\right) \end{aligned}$$

c. Evaluate $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$.

$$\begin{aligned} \sin^{-1}\left(\sin\left(\tfrac{2\pi}{3}\right)\right) &= \sin^{-1}\left(\tfrac{\sqrt{3}}{2}\right) && \left(\text{since } \sin\left(\tfrac{2\pi}{3}\right) = \tfrac{\sqrt{3}}{2}\right) \\ &= \tfrac{\pi}{3} && \left(\text{since } \sin\left(\tfrac{\pi}{3}\right) = \tfrac{\sqrt{3}}{2} \text{ and } -\tfrac{\pi}{2} \le \tfrac{\pi}{3} \le \tfrac{\pi}{2}\right) \end{aligned}$$