

SOLUTIONS: Worksheet 3

1. Find an algebraic rule (i.e., a “formula”) for the sinusoidal function f graphed in Figure 1.

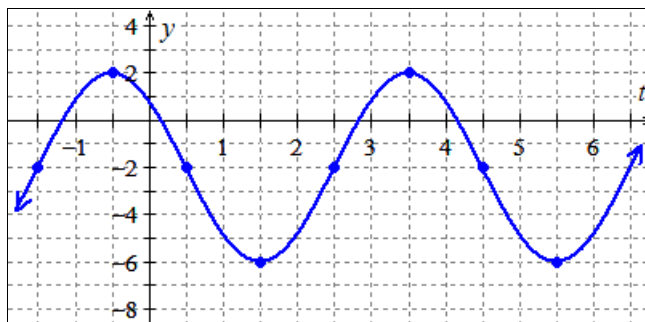


Figure 1: The graph of $y = f(t)$.

We can use the sine or cosine function to construct our rule: let's use cosine. So we can assume that f has form

$$f(t) = A \cos(\omega(t - h)) + k.$$

The amplitude of $y = f(t)$ is 4, so $|A| = 4$.

The period of $y = f(t)$ is $3.5 - (-0.5) = 4$, so we know that

$$\begin{aligned} 4 &= 2\pi \cdot \frac{1}{\omega} \\ \Rightarrow \omega &= \frac{2\pi}{4} = \frac{\pi}{2}. \end{aligned}$$

We can “start” a cosine wave at $t = -\frac{1}{2}$, so we can take $h = -\frac{1}{2}$.

Since the midline is $y = -2$, we know that $k = -2$

$$\text{Thus, } f(t) = 4 \cos\left(\frac{\pi}{2}\left(t - \left(-\frac{1}{2}\right)\right)\right) - 2 = 4 \cos\left(\frac{\pi}{2}\left(t + \frac{1}{2}\right)\right) - 2.$$

Other possible rules for f :

$$f(t) = -4 \cos\left(\frac{\pi}{2}\left(t - \frac{3}{2}\right)\right) - 2$$

$$f(t) = 4 \sin\left(\frac{\pi}{2}\left(t + \frac{3}{2}\right)\right) - 2$$

$$f(t) = -4 \sin\left(\frac{\pi}{2}\left(t - \frac{1}{2}\right)\right) - 2$$

(there are many other possibilities)

2. Find an algebraic rule (i.e., a “formula”) for the sinusoidal function g graphed in Figure 2.

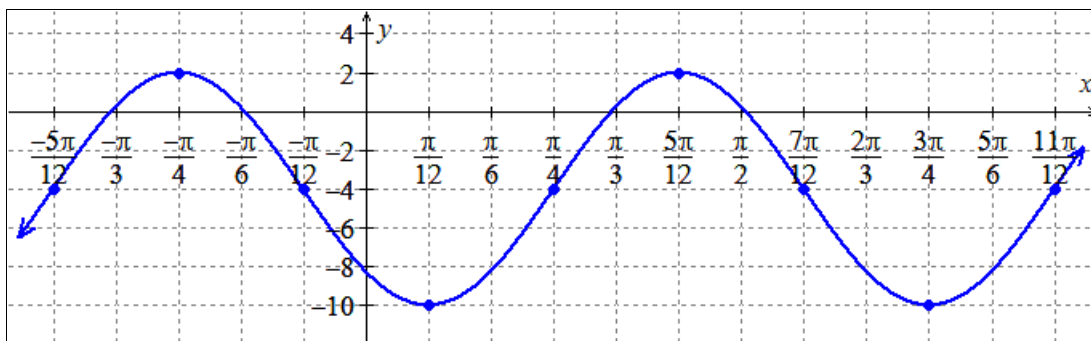


Figure 2: The graph of $y = g(x)$.

We can use the sine or cosine function to construct our rule: let's use sine. So we can assume that g has form

$$g(x) = A\sin(\omega(x - h)) + k.$$

The amplitude of $y = g(x)$ is 6, so $|A| = 6$.

The period of $y = g(x)$ is $\frac{5\pi}{12} - \left(-\frac{\pi}{4}\right) = \frac{2\pi}{3}$, so we know that

$$\begin{aligned}\frac{2\pi}{3} &= 2\pi \cdot \frac{1}{\omega} \\ \Rightarrow \omega &= 2\pi \cdot \frac{3}{2\pi} = 3.\end{aligned}$$

We can “start” a sine wave at $x = \frac{\pi}{4}$, so we can take $h = \frac{\pi}{4}$.

Since the midline is $y = -4$, we know that $k = -4$

Thus, $g(x) = 6\sin\left(3\left(x - \frac{\pi}{4}\right)\right) - 4$.

Other possible rules for g :

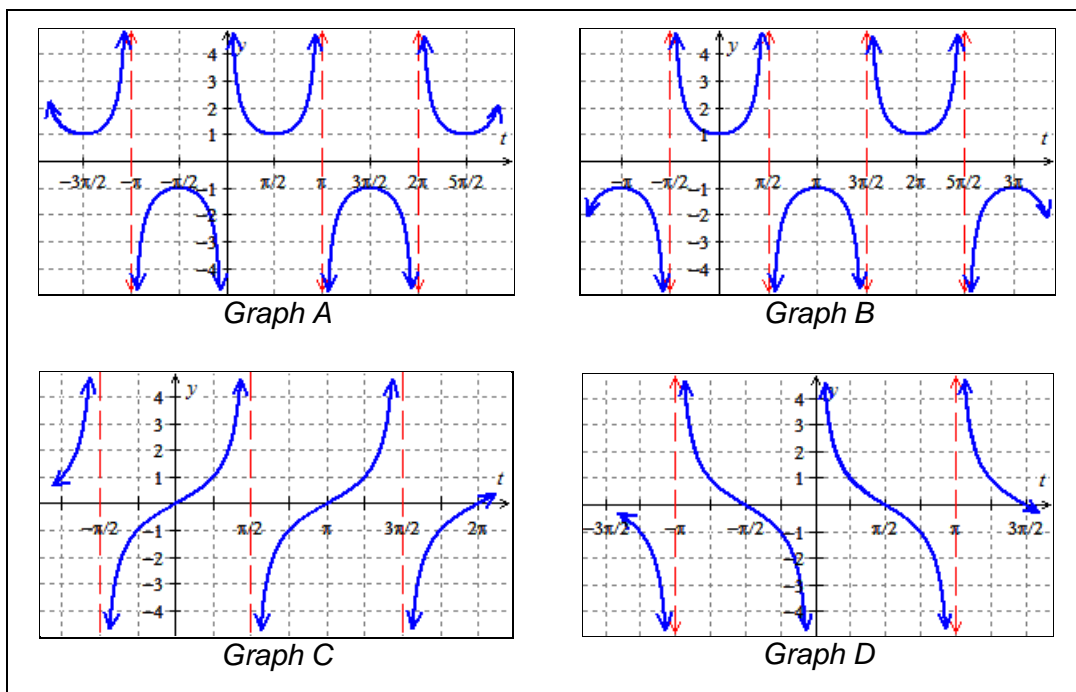
$$g(x) = -6\sin\left(3\left(x + \frac{\pi}{12}\right)\right) - 4$$

$$g(x) = 6\cos\left(3\left(x + \frac{\pi}{4}\right)\right) - 4$$

$$g(x) = -6\cos\left(3\left(x - \frac{\pi}{12}\right)\right) - 4$$

(there are many other possibilities)

3. Use the given graphs to answer questions below with the phrase “Graph A” or “Graph B” or “Graph C” or “Graph D”.



- a. Which graph represents $y = \sec(t)$?

Graph B represents $y = \sec(t)$.

- b. Which of graph represents $y = \cot(t)$? [1 point]

Graph D represents $y = \cot(t)$.