# MTH 112: Identities and Formulas Reference Sheet

This sheet will be provided to students during relevant exams.

### Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

### Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

### Double Angle Identities

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = 1 - 2\sin^2(A)$$

$$\cos(2A) = 2\cos^2(A) - 1$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

### Sum and Difference Identities

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$$

$$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$$

### Half Angle Identities

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos(A)}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 + \cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos(A)}{\sin(A)}$$

### Product-to-Sum Identities

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A - B) + \sin(A + B)\right]$$

### Conic Sections: Ellipses

IMPLICIT EQUATION:

$$1 = \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2}$$

PARAMETRIC SYSTEM:

$$\begin{cases} x = a\cos(t) + h \\ y = b\sin(t) + k \end{cases}$$

### Sum of Sine and Cosine Identity

$$A_1\sin(\omega t) + A_2\cos(\omega t) = A\sin(\omega t + \phi)$$

where $A = \sqrt{A_1^2 + A_2^2}$ and $\tan(\phi) = \frac{A_2}{A_1}$, and

$\phi$ satisfies $\cos(\phi) = \frac{A_1}{A}$ and $\sin(\phi) = \frac{A_2}{A}$

### Dot Product

$$\vec{v} \cdot \vec{w} = a_1 a_2 + b_1 b_2$$

### Angle Between Vectors

$$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|\|\vec{w}\|}$$

### Euler's Formula

$$re^{i\cdot\theta} = r\cos(\theta) + r\sin(\theta)\cdot i$$

### Sum-to-Product Identities

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$