Being Prepared for MTH 112 (and Calculus)

MTH 112 is the second course in a two-course sequence that starts with MTH 111. The MTH 111/112 sequence is often called "precalculus" since the content of the two courses is intended to prepare students for Calculus. We use the same textbook for both classes but we cover distinct components of the book in each course so, for example, if you never learned some of the topics in MTH 111 (or skip MTH 111 entirely via placement testing), even if you manage to succeed in MTH 112, you may struggle in Calculus since there might be "holes" in your understanding of the prerequisite material. So, if you plan to take Calculus but are concerned about your understanding of the prerequisite material, it's important that you review that math before you take Calculus; one way you can review the topics studied in MTH 111 is by studying my MTH 111 Online Lecture Notes:

https://spot.pcc.edu/~phaberma/MTH_111/Lecture_Notes/MTH111_Lecture_Notes.html

Some of the topics that are studied in MTH 111 aren't "that" important in MTH 112 (so it's not as important that you review them immediately) but other topics in MTH 111 are crucial for understanding what we'll study in MTH 112 so it's important that you review them immediately, especially if it's been awhile since you took MTH 111. In my MTH 111 Online Lecture Notes, the topics in *Section I: Functions and Their Graphs* are most important for MTH 112, especially *Unit 6: Inverse Functions* and *Units 7-9: Graph Transformations*.

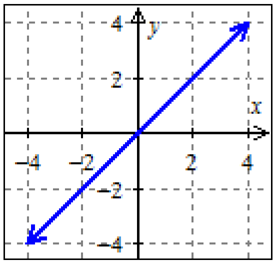
Included in this packet is a list of "back-pocket functions" (see below) and a two-page document called "Sets and Numbers" that reviews some basic set-notation that's important to be familiar with.

Back-Pocket Functions

In the next couple of weeks we will study how we can create new functions from old ones. But the only way we can do this is if we are already familiar with some "old functions". The ten functions graphed on the back of this page are some of the most important functions to be familiar with since they are often the building-blocks that we use to construct more complicated functions. I call these functions "back-pocket functions" since I think that we should always carry their graphs with us in our figurative back-pockets, i.e., you should be intimately familiar with these functions and their graphs! (You can find the domains and ranges of most of these functions on pages 80-82 of our textbook.)

Back-Pocket Functions

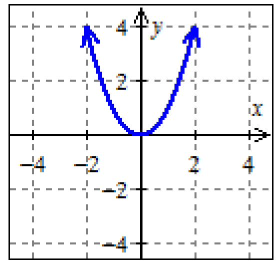
a. $f(x) = x$



Domain:

Range:

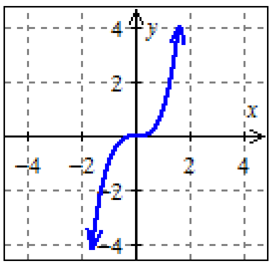
b. $f(x) = x^2$



Domain:

Range:

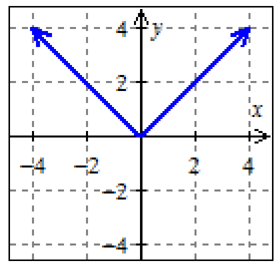
c. $f(x) = x^3$



Domain:

Range:

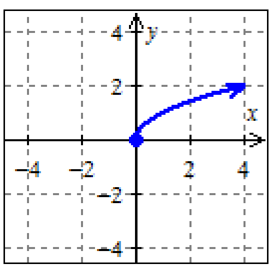
d. $f(x) = |x|$



Domain:

Range:

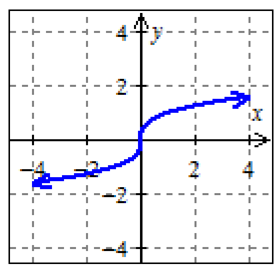
e. $f(x) = \sqrt{x}$



Domain:

Range:

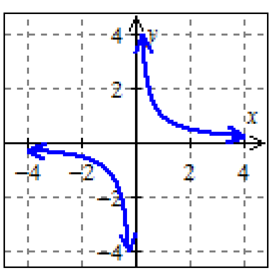
f. $f(x) = \sqrt[3]{x}$



Domain:

Range:

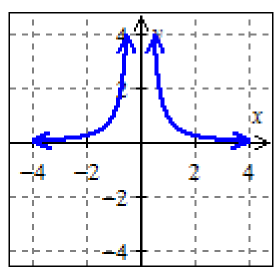
g. $f(x) = \dfrac{1}{x}$



Domain:

Range:

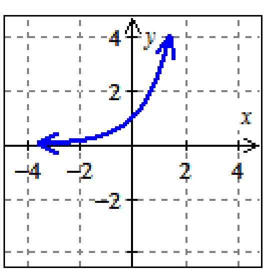
h. $f(x) = \dfrac{1}{x^2}$



Domain:

Range:

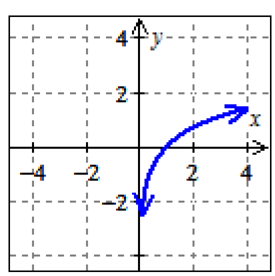
i. $f(x) = e^x$



Domain:

Range:

j. $f(x) = \ln(x)$



Domain:

Range:

Sets and Numbers

DEFINITION: A **set** is a collection of objects specified in a manner that enables one to determine if a given object is or is not in the set. In other words, a set is a well-defined collection of objects.

Roster Notation involves listing the elements in a set within *curly brackets*: “{ }”.

DEFINITION: An object in a set is called an **element** of the set. (symbol: “$\in$”)

EXAMPLE: 5 is an element of the set $\{4, 5, 6, 7, 8, 9\}$. We can express this symbolically:

$$5 \in \{4, 5, 6, 7, 8, 9\}$$

DEFINITION: The **empty set**, denoted Ø, is the set with no elements.

Note that $0 \neq \varnothing$. “0” represents the number of elements in the empty set but “$\varnothing$” is a set, not a number.

EXAMPLE: All of the whole numbers (positive and negative) form a set. This set is called the **integers**, and is represented by the symbol $\mathbb{Z}$. We can express the set of integers in roster notation:

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

The symbol $\mathbb{Z}$ is used to represent the integers because the German word for “counting” is *zahlen* and the (positive) integers are used for counting.

Now that we have the integers, we represent sets like “All of the whole numbers between 3 and 10” using **set-builder notation**:

$$\text{"All the whole numbers between 3 and 10"} = \{x \mid x \in \mathbb{Z} \text{ and } 3 < x < 10\}$$

This vertical line means "*such that*"

Armed with set-builder notation, we can define some important **sets of numbers**:

DEFINITIONS: The set of **natural numbers**: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

The set of **integers**: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

The set of **rational numbers**: $\mathbb{Q} = \left\{x \,\middle|\, x = \frac{p}{q} \text{ and } p, q \in \mathbb{Z} \text{ and } q \neq 0\right\}$

The set of **real numbers**: $\mathbb{R}$ (All the numbers on the number line.)

The set of **complex numbers**: $\mathbb{C} = \left\{x \,\middle|\, x = a + bi \text{ and } a, b \in \mathbb{R} \text{ and } i = \sqrt{-1}\right\}$

Usually we will assume that the number-set in question is the set of real numbers, $\mathbb{R}$, unless we are specifically asked to consider an alternative set.

Since we use the real numbers so often, we have special notation for subsets of the real numbers: **interval notation**. Interval notation involves square or round brackets. Use the examples below to understand how interval notation works. Note that when the interval has no upper (or lower) bound, the symbol ∞ (or $-\infty$) is used; see **d** and **e** below.

EXAMPLE:

a. $\{x \,|\, x \in \mathbb{R} \text{ and } -2 \leq x \leq 3\} = [-2, 3]$

$\uparrow$ Set-builder Notation $\qquad \uparrow$ Interval Notation

We use square brackets here since the endpoints are included

b. $\{x \,|\, x \in \mathbb{R} \text{ and } -2 < x < 3\} = (-2, 3)$

$\uparrow$ Set-builder Notation $\qquad \uparrow$ Interval Notation

We use round brackets here since the endpoints are NOT included.

c. $\{x \,|\, x \in \mathbb{R} \text{ and } -2 < x \leq 3\} = (-2, 3]$

$\uparrow$ Set-builder Notation $\qquad \uparrow$ Interval Notation

We use a round bracket on the left since -2 is NOT included.

d. $\{x \,|\, x \in \mathbb{R} \text{ and } x \leq 4\} = (-\infty, 4]$

$\uparrow$ Set-builder Notation $\qquad \uparrow$ Interval Notation

We ALWAYS use a round bracket with $-\infty$ since it is NOT a number in the set.

e. $\{x \,|\, x \in \mathbb{R} \text{ and } x \geq 4\} = [4, \infty)$

$\uparrow$ Set-builder Notation $\qquad \uparrow$ Interval Notation

We ALWAYS use a round bracket with ∞ since it is NOT a number in the set.