Section I: The Trigonometric Functions

Chapter 8: Graphs of the Other Trigonometric Functions

Recall from Chapter 4 the following definitions of the “other trigonometric functions”:

DEFINITIONS: The **tangent function**, denoted $\tan(\theta)$, is defined by $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$.

The **cotangent function**, denoted $\cot(\theta)$, is defined by $\cot(t) = \dfrac{1}{\tan(\theta)}$.

Consequently, $\cot(t) = \dfrac{\cos(\theta)}{\sin(\theta)}$.

The **secant function**, denoted $\sec(\theta)$, is defined by $\sec(\theta) = \dfrac{1}{\cos(\theta)}$.

The **cosecant function**, denoted $\csc(\theta)$, is defined by $\csc(\theta) = \dfrac{1}{\sin(\theta)}$.

In this chapter, we'll graph all four of these functions. [In §5.5 of our textbook, transformations of these functions are studied but you are only responsible for understanding the non-transformed versions of these functions.]

EXAMPLE 1: Graph the *tangent* function, $y = \tan(t)$.

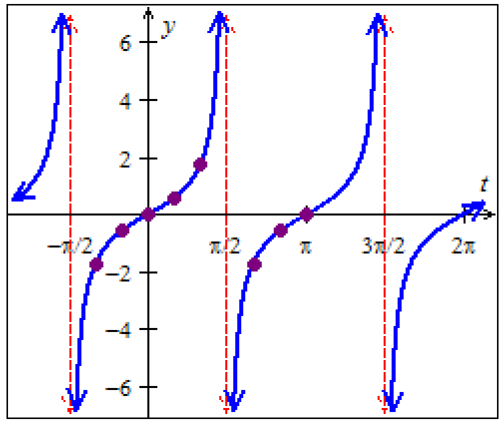
SOLUTION:

We can use the fact that

$$\tan(t) = \frac{\sin(t)}{\cos(t)}$$

to calculate values of tangent, and we can use these values to form ordered pairs that we can plot on a coordinate plane. See the table and graph in Figure 1 on the next page:

t	$\tan(t)$	$(t, \tan(t))$
$-\frac{\pi}{2}$	undefined	no point
$-\frac{\pi}{3}$	$-\sqrt{3}$	$\left(-\frac{\pi}{3}, -\sqrt{3}\right)$
$-\frac{\pi}{6}$	$-\frac{1}{\sqrt{3}}$	$\left(-\frac{\pi}{6}, -\frac{1}{\sqrt{3}}\right)$
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}}$	$\left(\frac{\pi}{6}, \frac{1}{\sqrt{3}}\right)$
$\frac{\pi}{3}$	$\sqrt{3}$	$\left(\frac{\pi}{3}, \sqrt{3}\right)$
$\frac{\pi}{2}$	undefined	no point
$\frac{2\pi}{3}$	$-\sqrt{3}$	$\left(\frac{2\pi}{3}, -\sqrt{3}\right)$
$\frac{5\pi}{6}$	$-\frac{1}{\sqrt{3}}$	$\left(\frac{5\pi}{6}, -\frac{1}{\sqrt{3}}\right)$
π	0	$(\pi, 0)$



Graph of $y = \tan(t)$.

Figure 1

Note that the period of the tangent function is π units, unlike the sine and cosine functions whose periods are both 2π units.

Also note that $\tan(t)$ is undefined for $t = -\frac{\pi}{2}$, $t = \frac{\pi}{2}$, $t = \frac{3\pi}{2}$, etc., since $\cos(t)$ equals 0 at these values and division by 0 is undefined. As you can see in the graph, $y = \tan(t)$ has a *vertical asymptote* at $t = -\frac{\pi}{2}$, $t = \frac{\pi}{2}$, $t = \frac{3\pi}{2}$, etc., i.e., $y = \tan(t)$ has a vertical asymptote at each t-value that makes $\cos(t) = 0$. The fact that there are vertical asymptotes tells us that there are no maximum or minimum outputs for the tangent function. Since the definitions of *midline* and *amplitude* involve maximum and minimum outputs, the tangent function has no midline or amplitude. Still, the t-axis (i.e., the line $y = 0$) behaves like a midline for $y = \tan(t)$.

EXAMPLE 2: Graph the *secant* function, $y = \sec(t)$.

SOLUTION:

We can use the fact that

$$\sec(t) = \frac{1}{\cos(t)}$$

to help us graph $y = \sec(t)$. In Figure 2, we've graphed $y = \sec(t)$ on a coordinate plane that also shows the graph of $y = \cos(t)$. Notice that $y = \sec(t)$ has vertical asymptotes at all t-values for which $\cos(t) = 0$.

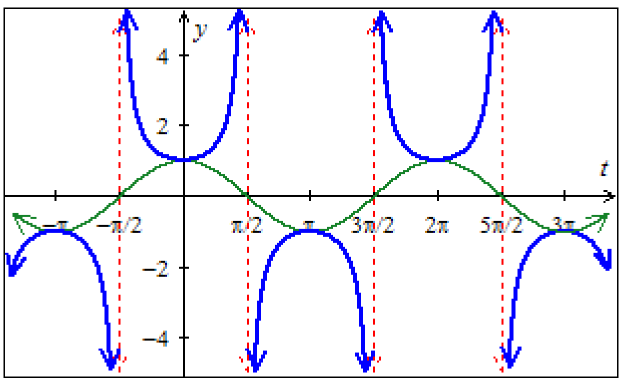


Figure 2: The graph of $y = \sec(t)$ (in blue) and $y = \cos(t)$ (in green)

EXAMPLE 3: Graph the *cosecant* function, $y = \csc(t)$.

SOLUTION:

We can use the fact that

$$\csc(t) = \frac{1}{\sin(t)}$$

to help us graph $y = \csc(t)$. In Figure 3, we've graphed $y = \csc(t)$ on a coordinate plane that also shows the graph of $y = \sin(t)$. Notice that $y = \csc(t)$ has vertical asymptotes at all t-values for which $\sin(t) = 0$.

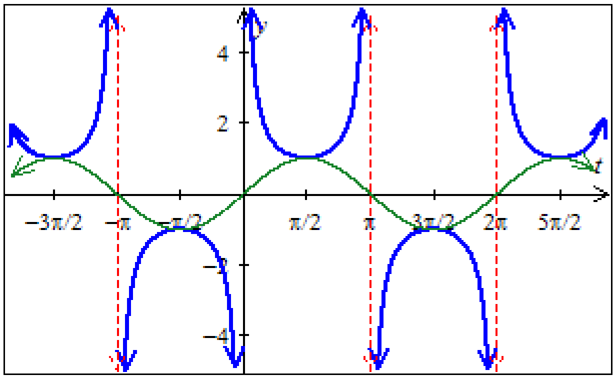


Figure 3: The graph of $y = \csc(t)$ (in blue) and $y = \sin(t)$ (in green)

EXAMPLE 4: Graph the *cotangent* function, $y = \cot(t)$.

SOLUTION:

We can use the fact that

$$\cot(t) = \frac{1}{\tan(t)}$$

to help us graph $y = \cot(t)$. In Figure 4, we've graphed $y = \cot(t)$ on a coordinate plane; notice that $y = \cot(t)$ has vertical asymptotes at all t-values for which $\tan(t) = 0$.

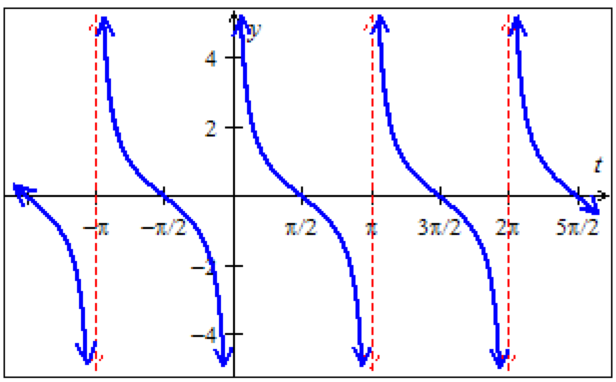


Figure 4: The graph of $y = \cot(t)$.