Section I: The Trigonometric Functions

Chapter 3, Part 2: Intro to the Trigonometric Functions

As mentioned in Part 1 of Chapter 3, there are four other trigonometric functions. These four functions are defined in terms of the sine and cosine functions:

DEFINITIONS:

The **tangent function**, denoted $\tan(\theta)$, is defined by $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$.

The **cotangent function**, denoted $\cot(\theta)$, is defined by $\cot(\theta) = \dfrac{1}{\tan(\theta)}$.

Consequently, $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$.

The **secant function**, denoted $\sec(\theta)$, is defined by $\sec(\theta) = \dfrac{1}{\cos(\theta)}$.

The **cosecant function**, denoted $\csc(\theta)$, is defined by $\csc(\theta) = \dfrac{1}{\sin(\theta)}$.

Since all three of these functions are defined in terms of the sine and cosine function, it's reasonable to consider the sine and cosine to be more important. The tangent function is the ratio of the sine and cosine functions so it provides truly different information than does sine or cosine, so the tangent function is also important. But cotangent, secant, and cosecant are just reciprocals of tangent, cosine, and sine so they don't provide any new information (instead, they provide the same information differently) so, arguably, they are less important functions. In Chapter 5 we'll look at graphs of these "other trig functions" but in this chapter we'll focus on getting familiar with their definitions and, in the process, review the sine and cosine values we observed in Part 1 of Chapter 3.

EXAMPLE 1: Find $\tan(\theta)$, $\sec(\theta)$, $\csc(\theta)$, and $\cot(\theta)$ if...

a. $\ldots\theta = \pi$. **b.** $\ldots\theta = \frac{3\pi}{2}$. **c.** $\ldots\theta = 2\pi$.

SOLUTION:

a. $\tan(\pi) = \dfrac{\sin(\pi)}{\cos(\pi)}$ (using the definition of tangent)

$= \dfrac{0}{-1}$ (since $\sin(\pi) = 0$ and $\cos(\pi) = -1$; see Part 1 of Chapter 3)

$= 0$

$\cot(\pi) = \dfrac{\cos(\pi)}{\sin(\pi)}$ (using the definition of cotantent)

Recall from Part 1 of Chapter 3 that $\sin(\pi) = 0$ so $\cot(\pi)$ involves division by 0, so $\cot(\pi)$ is *undefined*.

$\sec(\pi) = \dfrac{1}{\cos(\pi)}$ (using the definition of secant)

$= \dfrac{1}{-1}$ (since $\cos(\pi) = -1$; see Part 1 of Chapter 3)

$= -1$

$\csc(\pi) = \dfrac{1}{\sin(\pi)}$ (using the definition of cosecant)

Since $\sin(\pi) = 0$, $\csc(\pi)$ involves division by 0 so $\csc(\pi)$ is *undefined*.

b. $\tan\left(\frac{3\pi}{2}\right) = \dfrac{\sin\left(\frac{3\pi}{2}\right)}{\cos\left(\frac{3\pi}{2}\right)}$ (using the definition of tangent)

Recall from Part 1 of Chapter 3 that $\cos\left(\frac{3\pi}{2}\right) = 0$ so $\tan\left(\frac{3\pi}{2}\right)$ involves division by 0, so $\tan\left(\frac{3\pi}{2}\right)$ is *undefined*.

$\cot\left(\frac{3\pi}{2}\right) = \dfrac{\cos\left(\frac{3\pi}{2}\right)}{\sin\left(\frac{3\pi}{2}\right)}$ (using the definition of cotangent)

$= \dfrac{0}{-1}$ (since $\cos\left(\frac{3\pi}{2}\right) = 0$ and $\sin\left(\frac{3\pi}{2}\right) = -1$; see Part 1 of Chapter 3)

$= 0$

$$\sec\left(\frac{3\pi}{2}\right) = \frac{1}{\cos\left(\frac{3\pi}{2}\right)}$$ (using the definition of secant)

Since $\cos\left(\frac{3\pi}{2}\right) = 0$, $\sec\left(\frac{3\pi}{2}\right)$ involves division by 0 so $\sec\left(\frac{3\pi}{2}\right)$ is *undefined*.

$$\csc\left(\frac{3\pi}{2}\right) = \frac{1}{\sin\left(\frac{3\pi}{2}\right)}$$ (using the definition of cosecant)

$$= \frac{1}{-1}$$ (since $\sin\left(\frac{3\pi}{2}\right) = -1$; see Part 1 of Chapter 3)

$$= -1$$

c. $$\tan(2\pi) = \frac{\sin(2\pi)}{\cos(2\pi)}$$ (using the definition of tangent)

$$= \frac{0}{1}$$ (since $\sin(2\pi) = 0$ and $\cos(2\pi) = 1$; see Part 1 of Chapter 3)

$$= 0$$

$$\cot(2\pi) = \frac{\cos(2\pi)}{\sin(2\pi)}$$ (using the definition of cotangent)

Recall from Part 1 of Chapter 3 that $\sin(2\pi) = 0$ so $\csc(2\pi)$ involves division by 0, so $\cot(2\pi)$ is *undefined*.

$$\sec(2\pi) = \frac{1}{\cos(2\pi)}$$ (using the definition of secant)

$$= \frac{1}{1}$$ (since $\cos(2\pi) = 1$; see Part 1 of Chapter 3)

$$= 1$$

$$\csc(2\pi) = \frac{1}{\sin(2\pi)}$$ (using the definition of cosecant)

Since $\sin(2\pi) = 0$, $\cot(2\pi)$ involves division by 0 so $\csc(2\pi)$ is *undefined*.

Recall the Pythagorean Identity from Part 1 of Chapter 3: $\sin^2(\theta) + \cos^2(\theta) = 1$. There are two other identities that can be obtained from the Pythagorean Identity.

One of these identities can be found by dividing both sides of the Pythagorean identity by $\cos^2(\theta)$:

$$\begin{aligned} & \sin^2(\theta) + \cos^2(\theta) = 1 \\ \Rightarrow\ & \frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)} \\ \Rightarrow\ & \tan^2(\theta) + 1 = \sec^2(\theta) \end{aligned}$$

Alternatively, we can divide both sides of the Pythagorean identity by $\sin^2(\theta)$ and find another identity:

$$\begin{aligned} & \sin^2(\theta) + \cos^2(\theta) = 1 \\ \Rightarrow\ & \frac{\sin^2(\theta)}{\sin^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)} \\ \Rightarrow\ & 1 + \cot^2(\theta) = \csc^2(\theta) \end{aligned}$$

This gives us three identities that are considered “The Pythagorean Identities”.

The Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$