Section II: Trigonometric Identities

Chapter 1: Review of Identities

Recall the following definition that we first studied in Section I: Chapter 3:

DEFINITION: An **identity** is an equation that is true for all values in the domains of the involved expressions.

In this chapter, we'll list the identities that we studied in Section I. If you haven't yet learned these identities, you should learn (i.e., understand and memorize) all of them.

In Section I, Chapter 3, we noticed the following identities:

Some Identities

$$\sin(\theta) = \sin(\theta + 2\pi)$$

$$\cos(\theta) = \cos(\theta + 2\pi)$$

$$\cos(\theta) = \sin\left(\theta + \frac{\pi}{2}\right)$$

$$\sin(\theta) = \cos\left(\theta - \frac{\pi}{2}\right)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\sin(\theta) = \sin(\pi - \theta)$$

Also in Section I, Chapter 3, we defined the "other" trigonometric functions and obtained identities in the process:

Other Trig Functions

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

Also in Section I, Chapter 3, we discovered the Pythagorean identities:

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Sometimes we need to use different forms of the Pythagorean Identities. By subtracting expressions from both sides of the different Pythagorean Identities, we can obtain the following identities. Be sure to convince yourself that these are all valid by adjusting the appropriate Pythagorean Identities.

More Pythagorean Identities

$$\sin^2(\theta) = 1 - \cos^2(\theta)$$

$$\cos^2(\theta) = 1 - \sin^2(\theta)$$

$$\tan^2(\theta) = \sec^2(\theta) - 1$$

$$\sec^2(\theta) - \tan^2(\theta) = 1$$

$$\cot^2(\theta) = \csc^2(\theta) - 1$$

$$\csc^2(\theta) - \cot^2(\theta) = 1$$