Extra Practice for Section IV: Chapter 2

1. Suppose that $\vec{v} = \langle -3, 7 \rangle$ and $\vec{w} = \langle 2, 10 \rangle$.

a. Find $\vec{v} \cdot \vec{w}$.

b. Find the angle between $\vec{v}$ and $\vec{w}$.

Click here to see the solution to **1.**

2. Suppose that $\vec{m} = 7\vec{i} - 4\vec{j}$ and $\vec{n} = -5\vec{i} - 2\vec{j}$.

a. Find $\vec{m} \cdot \vec{n}$.

b. Find the angle between $\vec{m}$ and $\vec{n}$.

Click here to see the solution to **2.**

Solution to **1.**

1. Suppose that $\vec{v} = \langle -3, 7 \rangle$ and $\vec{w} = \langle 2, 10 \rangle$.

a. Find $\vec{v} \cdot \vec{w}$.

$$\begin{aligned} \vec{v} \cdot \vec{w} &= \langle -3, 7 \rangle \cdot \langle 2, 10 \rangle \\ &= (-3)(2) + (7)(10) \\ &= -6 + 70 \\ &= 64 \end{aligned}$$

b. Find the angle between $\vec{v}$ and $\vec{w}$.

We can use the identity $\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cos(\theta)$, where θ is the angle between vectors $\vec{v}$ and $\vec{w}$. Above in **1.a**. we discovered that $\vec{v} \cdot \vec{w} = 64$, and in **1.b.** of Extra Practice for Section IV: Chapter 1 we determined that $\|\vec{v}\| = \sqrt{58}$ and $\|\vec{w}\| = 2\sqrt{26}$. Now we can substitute all of these values into the identity and find the angle between the vectors:

$$\begin{aligned} & \vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos(\theta) \\ \Rightarrow \quad & 64 = \sqrt{58} \cdot 2\sqrt{26} \cos(\theta) \\ \Rightarrow \quad & \cos(\theta) = \frac{64}{2\sqrt{58} \cdot \sqrt{26}} \\ \Rightarrow \quad & \theta = \cos^{-1}\left(\frac{32}{\sqrt{58} \cdot \sqrt{26}} \right) \approx 34.51^\circ \end{aligned}$$

So the angle between vectors $\vec{v}$ and $\vec{w}$ is about 34.51°.

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Solution to **2.**

2. Suppose that $\vec{m} = 7\vec{i} - 4\vec{j}$ and $\vec{n} = -5\vec{i} - 2\vec{j}$.

a. Find $\vec{m} \cdot \vec{n}$.

$$\begin{aligned} \vec{m} \cdot \vec{n} &= \left(7\vec{i} - 4\vec{j}\right) \cdot \left(-5\vec{i} - 2\vec{j}\right) \\ &= (7)(-5) + (-4)(-2) \\ &= -27 \end{aligned}$$

b. Find the angle between $\vec{m}$ and $\vec{n}$.

We can use the identity $\vec{m} \cdot \vec{n} = \|\vec{m}\|\|\vec{n}\|\cos(\theta)$, where θ is the angle between vectors $\vec{m}$ and $\vec{n}$. Above in **2.a**. we discovered that $\vec{m} \cdot \vec{n} = -27$, and in **2.b.** of Extra Practice for Section IV: Chapter 1 we determined that $\|\vec{m}\| = \sqrt{65}$ and $\|\vec{n}\| = \sqrt{29}$. Now we can substitute all of these values into the identity and find the angle between the vectors:

$$\begin{aligned} & \vec{m} \cdot \vec{n} = \|\vec{m}\|\|\vec{n}\|\cos(\theta) \\ \Rightarrow \quad & -27 = \sqrt{65} \cdot \sqrt{29}\,\cos(\theta) \\ \Rightarrow \quad & \cos(\theta) = \frac{-27}{\sqrt{65} \cdot \sqrt{29}} \\ \Rightarrow \quad & \theta = \cos^{-1}\left(\frac{-27}{\sqrt{65} \cdot \sqrt{29}}\right) \approx 128.45^{\circ} \end{aligned}$$

So the angle between vectors $\vec{m}$ and $\vec{n}$ is about 128.45°.

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