Extra Practice for Section III: Chapter 3

1. Find a polar form, $z = re^{i\theta}$, of the complex number $z = -3 - 3\sqrt{3}i$.

Click here to see the solution to **1.**

2. Find a polar form, $z = re^{i\theta}$, of the complex number $z = 2 - 2i$.

Click here to see the solution to **2.**

3. Express the complex number $z = 10e^{i\cdot\frac{11\pi}{6}}$ in the form $z = a + bi$.

Click here to see the solution to **3.**

Solution to **1.**

1. Find a polar form, $z = re^{i\theta}$, of the complex number $z = -3 - 3\sqrt{3}\,i$.

We can associate the complex number $z = -3 - 3\sqrt{3}\,i$ with the rectangular ordered pair $(-3, -3\sqrt{3})$, and then translate this ordered pair into polar coordinates (r, θ), and finally use this polar ordered pair to obtain the polar form $z = re^{i\theta}$. First, let's find r:

$$\begin{aligned} r &= \sqrt{(-3)^2 + (-3\sqrt{3})^2} \\ &= \sqrt{9 + 9\cdot 3} \\ &= 6. \end{aligned}$$

Now, let's find θ:

$$\tan(\theta) = \frac{-3\sqrt{3}}{-3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\sqrt{3}\right) + \pi$$

(we add π since the given point is in quadrant 3 but the range of arctangent is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$)

$$\Rightarrow \quad \theta = \frac{4\pi}{3}$$

Thus, $z = 6e^{i\cdot\frac{4\pi}{3}}$ is a polar form of the complex number $z = -3 - 3\sqrt{3}\,i$.

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Solution to **2.**

2. Find a polar form, $z = re^{i\theta}$, of the complex number $z = 2 - 2i$.

We can associate the complex number $z = 2 - 2i$ with the rectangular ordered pair $(2, -2)$, and then translate this ordered pair into polar coordinates (r, θ), and finally use this polar ordered pair to obtain the polar form $z = re^{i\theta}$. First, let's find r:

$$\begin{aligned} r &= \sqrt{(2)^2 + (-2)^2} \\ &= \sqrt{4 + 4} \\ &= 2\sqrt{2}. \end{aligned}$$

Now, let's find θ:

$$\begin{aligned} & \tan(\theta) = \tfrac{-2}{2} \\ \Rightarrow \qquad & \theta = \tan^{-1}(-1) \\ \Rightarrow \qquad & \theta = -\tfrac{\pi}{4} \end{aligned}$$

Thus, $z = 2\sqrt{2}\, e^{i \cdot \left(-\frac{\pi}{4}\right)}$ is a polar form of the complex number $z = 2 - 2i$.

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Solution to **3.**

3. Express the complex number $z = 10e^{i\cdot\frac{11\pi}{6}}$ in the form $z = a + bi$.

$$\begin{aligned} z &= 10e^{i\cdot\frac{11\pi}{6}} \\ &= 10\cos\left(\tfrac{11\pi}{6}\right) + 10\sin\left(\tfrac{11\pi}{6}\right)\cdot i \\ &= 10\cdot\left(\tfrac{\sqrt{3}}{2}\right) + 10\cdot\left(-\tfrac{1}{2}\right)\cdot i \\ &= 5\sqrt{3} - 5i \end{aligned}$$

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