Extra Practice for Section III: Chapter 2

1. Find an equation involving polar coordinates r and θ whose graph is the same as each of the following Cartesian equations. (Isolate r in your conclusion.)

a. $y = -7x + 5$. **b.** $y = 3x^2$.

Click here to see the solution to **1.**

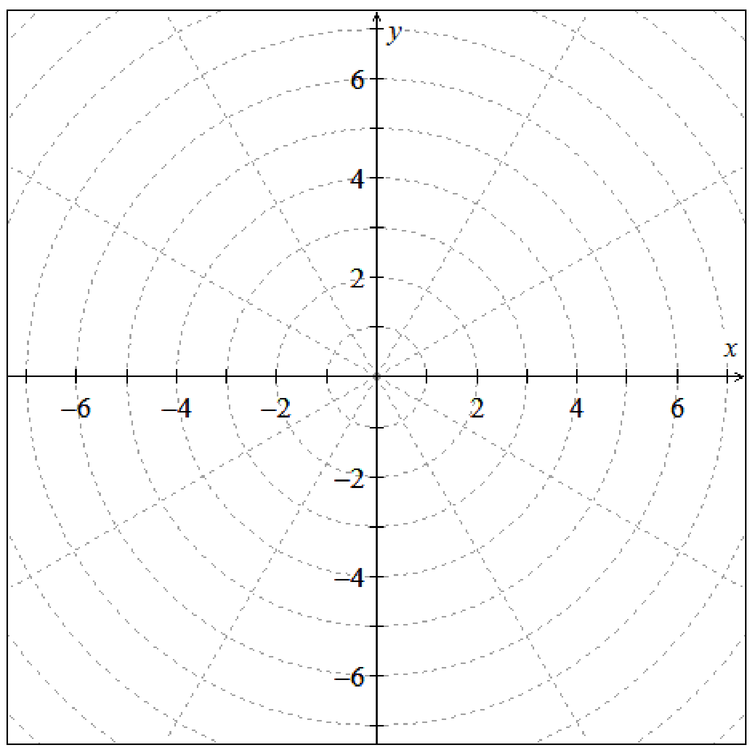
2. Find an equation involving rectangular coordinates x and y whose graph is equivalent to each of the following Polar equations. (Implicit equations are permitted.)

a. $r = 11\cos(\theta)$. **b.** $\theta = -\frac{\pi}{3}$.

Click here to see the solution to **2.**

3. Use multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$ to complete the first column in the table below and the determine the corresponding r-values if $r = 6\cos(2\theta)$ and then use the values in your table to sketch by hand a graph of $r = 6\cos(2\theta)$ on the polar plane below. (HINT: graph the function on your calculator so that you can predict the shape of the graph that you are trying to draw. Also, print the grid on paper and use a pencil to draw the graph.)

	θ	r
	0	
Quad. 1 angles		
	$\pi/2$	
Quad. 2 angles		
	π	
Quad. 3 angles		
	$3\pi/2$	
Quad. 4 angles		



Draw a graph of $r = 6\cos(2\theta)$ above.

Click here to see the solution to **3.**

Solution to **1.**

1. Find an equation involving polar coordinates r and θ whose graph is the same as each of the following Cartesian equations. (Isolate r in your conclusion.)

a. $y = -7x + 5$.

We can use the identities $x = r\cos(\theta)$ and $y = r\sin(\theta)$ to convert the rectangular coordinates x and y into the polar coordinates r and θ:

$$
\begin{aligned}
& y = -7x + 5 \\
\Rightarrow \quad & r\sin(\theta) = -7 \cdot r\cos(\theta) + 5 \quad (\text{since } x = r\cos(\theta) \text{ \& } y = r\sin(\theta)) \\
\Rightarrow \quad & r\sin(\theta) + 7r\cos(\theta) = 5 \\
\Rightarrow \quad & r\left(\sin(\theta) + 7\cos(\theta)\right) = 5 \\
\Rightarrow \quad & r = \frac{5}{\sin(\theta) + 7\cos(\theta)}
\end{aligned}
$$

So the equation $r = \frac{5}{\sin(\theta) + 7\cos(\theta)}$ in polar coordinates is equivalent to the equation $y = -7x + 5$ in rectangular coordinates.

b. $y = 3x^2$.

We can use the identities $x = r\cos(\theta)$ and $y = r\sin(\theta)$ to convert the rectangular coordinates x and y into the polar coordinates r and θ:

$$
\begin{aligned}
& y = 3x^2 \\
\Rightarrow \quad & r\sin(\theta) = 3\left(r\cos(\theta)\right)^2 \quad (\text{since } x = r\cos(\theta) \text{ \& } y = r\sin(\theta)) \\
\Rightarrow \quad & r\sin(\theta) = 3r^2\cos^2(\theta) \\
\Rightarrow \quad & \sin(\theta) = 3r\cos^2(\theta) \\
\Rightarrow \quad & \frac{\sin(\theta)}{3\cos^2(\theta)} = r \\
\Rightarrow \quad & r = \frac{1}{3} \cdot \frac{\sin(\theta)}{\cos(\theta)} \cdot \frac{1}{\cos(\theta)} \\
\Rightarrow \quad & r = \frac{1}{3} \cdot \tan(\theta) \cdot \sec(\theta)
\end{aligned}
$$

Therefore, the equation $r = \frac{1}{3}\tan(\theta)\sec(\theta)$ in polar coordinates is equivalent to the equation $y = 3x^2$ in rectangular coordinates.

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Solution to **2.**

2. Find an equation involving rectangular coordinates x and y whose graph is equivalent to each of the following Polar equations. (Implicit equations are permitted.)

a. $r = 11\cos(\theta)$.

We can use the identities $r = \sqrt{x^2 + y^2}$ and $x = r\cos(\theta)$ to convert the polar coordinates r and θ into rectangular coordinates x and y:

$$\begin{aligned}
& r = 11\cos(\theta) \\
\Rightarrow\ & \sqrt{x^2 + y^2} = 11\cos(\theta) && \text{(since } r = \sqrt{x^2 + y^2}\text{)} \\
\Rightarrow\ & \sqrt{x^2 + y^2} = 11\cdot\frac{x}{r} && \text{(since } x = r\cos(\theta) \text{ implies that } \cos(\theta) = \tfrac{x}{r}\text{)} \\
\Rightarrow\ & \sqrt{x^2 + y^2} = 11\cdot\frac{x}{\sqrt{x^2 + y^2}} && \text{(again using } r = \sqrt{x^2 + y^2}\text{)} \\
\Rightarrow\ & x^2 + y^2 = 11x
\end{aligned}$$

Thus, the equation $x^2 + y^2 = 11x$ in rectangular coordinates is equivalent to the equation $r = 11\cos(\theta)$ in polar coordinates.

b. $\theta = -\frac{\pi}{3}$.

We can use the identity $\tan(\theta) = \frac{y}{x}$ to convert the polar coordinates θ into rectangular coordinates x and y:

$$\begin{aligned}
& \theta = -\frac{\pi}{3} \\
\Rightarrow\ & \tan(\theta) = \tan\left(-\frac{\pi}{3}\right) && \text{(apply the tangent function to both sides)} \\
\Rightarrow\ & \tan(\theta) = -\sqrt{3} \\
\Rightarrow\ & \frac{y}{x} = -\sqrt{3}\,. && \text{(use the fact that } \tan(\theta) = \tfrac{y}{x}\text{)} \\
\Rightarrow\ & y = -\sqrt{3}\,x
\end{aligned}$$

So the equation $y = -\sqrt{3}\,x$ in rectangular coordinates is equivalent to the equation $\theta = -\frac{\pi}{3}$ in polar coordinates.

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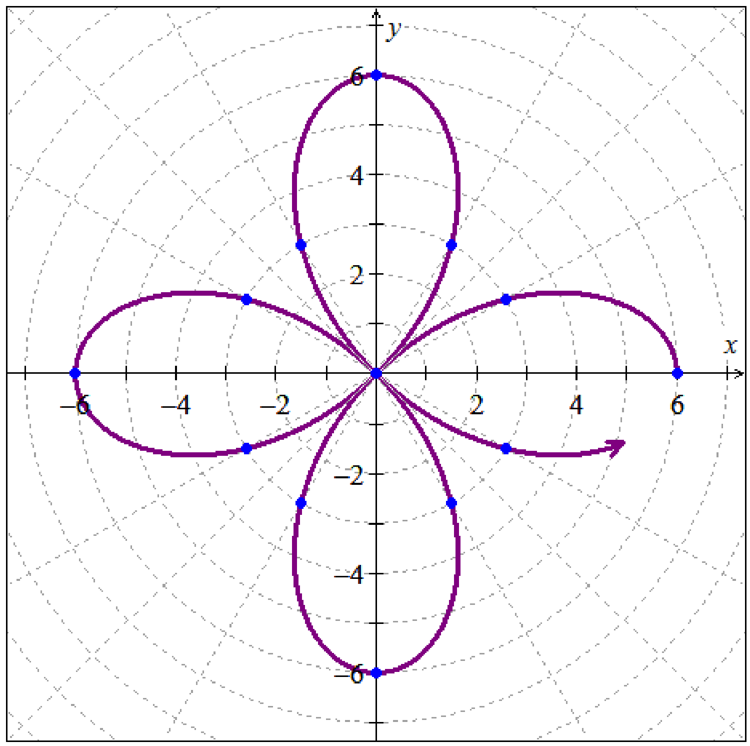
Solution to **3.**

3. Use multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$ to complete the first column in the table below and the determine the corresponding r-values if $r = 6\cos(2\theta)$ and then use the values in your table to sketch by hand a graph of $r = 6\cos(2\theta)$ on the polar plane below. (HINT: graph the function on your calculator so that you can predict the shape of the graph that you are trying to draw. Also, print the grid on paper and use a pencil to draw the graph.)

	$\boldsymbol{\theta}$	$\boldsymbol{r}$
	0	6
Quad. 1 angles	$\frac{\pi}{6}$	3
	$\frac{\pi}{4}$	0
	$\frac{\pi}{3}$	–3
	$\pi/2$	–6
Quad. 2 angles	$\frac{2\pi}{3}$	–3
	$\frac{3\pi}{4}$	0
	$\frac{5\pi}{6}$	3
	π	6
Quad. 3 angles	$\frac{7\pi}{6}$	3
	$\frac{5\pi}{4}$	0
	$\frac{4\pi}{3}$	–3
	$3\pi/2$	–6
Quad. 4 angles	$\frac{5\pi}{3}$	–3
	$\frac{7\pi}{4}$	0
	$\frac{11\pi}{6}$	3



A graph of $r = 6\cos(2\theta)$.

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