Extra Practice for Section III: Chapter 1

1. Translate each of the following rectangular coordinates $(x,\ y)$ into polar coordinates $(r,\ \theta)$. Your answers should be ordered pairs involving exact values, with θ in radians.

a. $(x,\ y) = (-4,\ -4)$

b. $(x,\ y) = \left(6,\ -6\sqrt{3}\right)$

Click here to see the solution to **1.**

2. Translate each of the following rectangular coordinates $(x,\ y)$ into polar coordinates $(r,\ \theta)$. Your answers should be ordered pairs involving approximations of θ in radians.

a. $(x,\ y) = (10,\ -2)$

b. $(x,\ y) = (-3,\ 7)$

Click here to see the solution to **2.**

3. Translate each of the following polar coordinates $(r,\ \theta)$ into rectangular coordinates $(x,\ y)$. Your answers should be ordered pairs involving exact values.

a. $(r,\ \theta) = \left(5,\ \frac{2\pi}{3}\right)$

b. $(r,\ \theta) = \left(16,\ 210^\circ\right)$

Click here to see the solution to **3.**

4. Translate each of the following polar coordinates $(r,\ \theta)$ into rectangular coordinates $(x,\ y)$. Your answers should be ordered pairs involving approximate values.

a. $(r,\ \theta) = \left(3,\ 80^\circ\right)$

b. $(r,\ \theta) = \left(7,\ -\frac{\pi}{10}\right)$

Click here to see the solution to **4.**

Solution to **1.**

1. Translate each of the following rectangular coordinates $(x,\ y)$ into polar coordinates $(r,\ \theta)$. Your answers should be ordered pairs involving exact values, with θ in radians.

a. $(x,\ y) = (-4,\ -4)$.

We can use the Pythagorean Theorem to find r:

$$\begin{aligned} r^2 &= (-4)^2 + (-4)^2 \\ \Rightarrow r^2 &= 16 + 16 \\ \Rightarrow \quad r &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

We can use tangent to find θ: $\tan(\theta) = \frac{-4}{-4} = 1$

We're familiar with the fact $\tan\left(\frac{\pi}{4}\right) = 1$ so we know that the angle should be a multiple of $\frac{\pi}{4}$. Since the given point is in the third quadrant, we can conclude that $\theta = \frac{5\pi}{4}$.

Thus, the rectangular coords $(-4,\ -4)$ are equivalent to the polar coords $\left(4\sqrt{2},\ \frac{5\pi}{4}\right)$.

b. $(x,\ y) = \left(6,\ -6\sqrt{3}\right)$.

We can use the Pythagorean Theorem to find r:

$$\begin{aligned} r^2 &= (6)^2 + \left(-6\sqrt{3}\right)^2 \\ \Rightarrow r^2 &= 36 + 36\cdot 3 \\ \Rightarrow \quad r &= \sqrt{144} = 12 \end{aligned}$$

We can use tangent to find θ: $\tan(\theta) = \frac{-6\sqrt{3}}{6} = -\sqrt{3}$

We're familiar with the fact $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ so we know that the angle is a multiple of $\frac{\pi}{3}$. Since the given point is in the fourth quadrant, we can conclude that $\theta = \frac{5\pi}{3}$. (Note that we could also use $\theta = -\frac{\pi}{3}$.)

So the rectangular coordinates $\left(-6,\ -6\sqrt{3}\right)$ are equivalent to the polar coords $\left(12,\ \frac{5\pi}{3}\right)$.

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Solution to **2.**

2. Translate each of the following rectangular coordinates (x, y) into polar coordinates (r, θ). Your answers should be ordered pairs involving approximations of θ in radians.

a. $(x, y) = (10, -2)$.

We can use the Pythagorean Theorem to find r:

$$r^2 = (10)^2 + (-2)^2$$
$$\Rightarrow \; r = \sqrt{104} = 2\sqrt{26}$$

We can use tangent to find θ: $\tan(\theta) = \frac{-2}{10}$

To solve this equation for θ we'll use arc-tangent. Since the given point is in the fourth quadrant and since the range of arc-tangent is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, we know that the output of arc tangent will be in the correct quadrant so we won't need to adjust it:

$$\theta = \tan^{-1}\left(-\tfrac{2}{10}\right) \approx -0.197$$

Thus, the rectangular coordinates $(10, -2)$ are approximately equivalent to the polar coordinates $\left(2\sqrt{26}, -0.197\right)$.

b. $(x, y) = (-3, 7)$.

We can use the Pythagorean Theorem to find r:

$$r^2 = (-3)^2 + (7)^2$$
$$\Rightarrow \; r = \sqrt{58}$$

We can use tangent to find θ: $\tan(\theta) = \frac{7}{-3}$

To solve this equation for θ we'll use arc-tangent. Since the given point is in the second quadrant but the range of arc-tangent is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, we know we'll have to add π to the output of arc tangent in order to put the angle into the correct quadrant:

$$\theta = \tan^{-1}\left(-\tfrac{7}{3}\right) + \pi \approx 1.98$$

Therefore, the rectangular coordinates $(-5, 2)$ are approximately equivalent to the polar coordinates $\left(\sqrt{58}, 1.98\right)$.

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Solution to **3.**

3. Translate each of the following polar coordinates (r, θ) into rectangular coordinates (x, y). Your answers should be ordered pairs involving exact values.

a. $(r, \theta) = \left(5, \frac{2\pi}{3}\right)$.

$$\begin{aligned} x &= r \cdot \cos(\theta) \\ &= 5 \cdot \cos\left(\tfrac{2\pi}{3}\right) \\ &= 5 \cdot \left(-\tfrac{1}{2}\right) \\ &= -\tfrac{5}{2} \end{aligned}$$

and

$$\begin{aligned} y &= r \cdot \sin(\theta) \\ &= 5 \cdot \sin\left(\tfrac{2\pi}{3}\right) \\ &= 5 \cdot \left(\tfrac{\sqrt{3}}{2}\right) \\ &= \tfrac{5\sqrt{3}}{2} \end{aligned}$$

So polar coordinates $\left(5, \frac{2\pi}{3}\right)$ are equivalent to rectangular coordinates $\left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$.

b. $(r, \theta) = \left(16, 210^\circ\right)$.

$$\begin{aligned} x &= r \cdot \cos(\theta) \\ &= 16 \cdot \cos\left(210^\circ\right) \\ &= 16 \cdot \left(-\tfrac{\sqrt{3}}{2}\right) \\ &= -8\sqrt{3} \end{aligned}$$

and

$$\begin{aligned} y &= r \cdot \sin(\theta) \\ &= 16 \cdot \sin\left(210^\circ\right) \\ &= 16 \cdot \left(-\tfrac{1}{2}\right) \\ &= -8 \end{aligned}$$

Thus, the polar coordinates $\left(16, 210^\circ\right)$ are equivalent to the rectangular coordinates $\left(-8\sqrt{3}, -8\right)$.

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Solution to **4.**

4. Translate each of the following polar coordinates (r, θ) into rectangular coordinates (x, y). Your answers should be ordered pairs involving approximate values.

a. $(r, \theta) = (3, 80^\circ)$.

$$\begin{aligned} x &= r \cdot \cos(\theta) \\ &= 3 \cdot \cos(80^\circ) \\ &\approx 0.52 \end{aligned} \qquad \text{and} \qquad \begin{aligned} y &= r \cdot \sin(\theta) \\ &= 3 \cdot \sin(80^\circ) \\ &\approx 2.9544 \end{aligned}$$

Therefore, the polar coordinates $(3, 80^\circ)$ are approximately equivalent to the rectangular coordinates $(0.52, 2.95)$.

b. $(r, \theta) = \left(7, -\frac{\pi}{10}\right)$.

$$\begin{aligned} x &= r \cdot \cos(\theta) \\ &= 7 \cdot \cos\left(-\tfrac{\pi}{10}\right) \\ &\approx 6.6574 \end{aligned} \qquad \text{and} \qquad \begin{aligned} y &= r \cdot \sin(\theta) \\ &= 7 \cdot \sin\left(-\tfrac{\pi}{10}\right) \\ &\approx -2.1631 \end{aligned}$$

Thus, the polar coordinates $\left(7, -\frac{\pi}{10}\right)$ are approximately equivalent to the rectangular coordinates $(6.6574, -2.1631)$.

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