Extra Practice for Section II: Chapter 4

These problems involve some identities that you don't need to memorize since they are listed on the "Identities and Formulas Reference Sheet" that I'll include with your Final Exam. You are encouraged to consult the "Identities and Formulas Reference Sheet" (provided here, at the end of this document) while you work on these problems so that you get familiar with using it.

1. Use a **sum-of-angles** or **difference-of-angles identity** to calculate the *exact value* of each of the following.

a. $\sin(165^\circ)$ Click here to see the solution to **1.a.**

b. $\cos\left(\frac{13\pi}{12}\right)$ Click here to see the solution to **1.b.**

c. $\tan\left(\frac{17\pi}{12}\right)$ Click here to see the solution to **1.c.**

Solution to **1.a.**

1.a. $\sin(165^\circ)$.

$$\begin{aligned}\sin(165^\circ) &= \sin(120^\circ + 45^\circ)\\ &= \sin(120^\circ)\cos(45^\circ) + \sin(45^\circ)\cos(120^\circ)\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\left(-\frac{1}{2}\right)\\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

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Solution to **1.b.**

1.b. $\cos\left(\frac{13\pi}{12}\right)$.

$$\begin{aligned}\cos\left(\frac{13\pi}{12}\right) &= \cos\left(\frac{3\pi}{12} + \frac{10\pi}{12}\right)\\ &= \cos\left(\frac{\pi}{4} + \frac{5\pi}{6}\right)\\ &= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{5\pi}{6}\right)\\ &= \frac{\sqrt{2}}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\ &= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\\ &= \frac{-\sqrt{6}-\sqrt{2}}{4} = -\frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

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Solution to **1.c.**

1.c. $\tan\left(\frac{17\pi}{12}\right)$

We could use the sum/difference formula for tangent but, assuming that we were trying to memorize the identities rather than copy them off of our Identities and Formulas Reference Sheet, it's not worth trying to memorize the relatively obscure identities for tangent since we can use what we know about sine and cosine along with the fact that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$:

$$
\begin{aligned}
\tan\left(\frac{17\pi}{12}\right) &= \frac{\sin\left(\frac{17\pi}{12}\right)}{\cos\left(\frac{17\pi}{12}\right)} \\
&= \frac{\sin\left(\frac{20\pi}{12} - \frac{3\pi}{12}\right)}{\cos\left(\frac{20\pi}{12} - \frac{3\pi}{12}\right)} \\
&= \frac{\sin\left(\frac{5\pi}{3} - \frac{\pi}{4}\right)}{\cos\left(\frac{5\pi}{3} - \frac{\pi}{4}\right)} \\
&= \frac{\sin\left(\frac{5\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{5\pi}{3}\right)}{\cos\left(\frac{5\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{5\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)} \\
&= \frac{-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right)\cdot\frac{\sqrt{2}}{2}} \\
&= \frac{\frac{-\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{2} - \sqrt{6}}{4}} \\
&= -\frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} - \sqrt{6}}\cdot\frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} + \sqrt{6}} \qquad \text{(we'll use the conjugate of the denominator to rationalize the denominator; this is not required.)} \\
&= -\frac{2 + 2\sqrt{12} + 6}{2 - 6} \\
&= -\frac{8 + 4\sqrt{3}}{-4} \\
&= 2 + \sqrt{3}
\end{aligned}
$$

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MTH 112: Identities and Formulas Reference Sheet

This sheet will be provided to students during the Final Exam.

Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

Double Angle Identities

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = 1 - 2\sin^2(A)$$

$$\cos(2A) = 2\cos^2(A) - 1$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

Sum and Difference Identities

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$$

$$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$$

Half Angle Identities

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos(A)}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 + \cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos(A)}{\sin(A)}$$

Product-to-Sum Identities

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A - B) + \sin(A + B)\right]$$

Conic Sections: Ellipses

IMPLICIT EQUATION: $$1 = \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2}$$

PARAMETRIC SYSTEM: $$\begin{cases} x = a\cos(t) + h \\ y = b\sin(t) + k \end{cases}$$

Sum of Sine and Cosine Identity

$$A_1\sin(\omega t) + A_2\cos(\omega t) = A\sin(\omega t + \phi)$$

where $A = \sqrt{A_1 + A_2}$ and $\tan(\phi) = \frac{A_2}{A_1}$, and

ϕ satisfies $\cos(\phi) = \frac{A_1}{A}$ and $\sin(\phi) = \frac{A_2}{A}$

Dot Product

$$\vec{v}\cdot\vec{w} = a_1 a_2 + b_1 b_2$$

$$\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos(\theta)$$

Sum-to-Product Identities

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Euler's Formula

$$re^{i\cdot\theta} = r\cos(\theta) + r\sin(\theta)\cdot i$$

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