Extra Practice for Section II: Chapter 3

1. Prove the following identities. (Be sure to organize your proof as shown in the Online Lecture Notes. This means that you should start your proof by writing one side of the identity and then use equal signs between equivalent expressions until you obtain the other side of the identity. You should only include one step on each line and you should align your equal signs on the left of each step. Compare your proofs with those given in the solutions to make sure that you are using the correct organization and technique.)

a. $\csc(t) - \sin(t) = \cot(t)\cos(t)$ Click here to see the solution to **1.a.**

b. $\tan^2(x)\sin^2(x) = \tan^2(x) - \sin^2(x)$ Click here to see the solution to **1.b.**

c. $\sec(\theta) + \tan(\theta) = \dfrac{\cos(\theta)}{1 - \sin(\theta)}$ Click here to see the solution to **1.c.**

d. $\dfrac{1}{1 - \cos(x)} - \dfrac{1}{1 + \cos(x)} = 2\cot(x)\csc(x)$ Click here to see the solution to **1.d.**

e. $\cot(A) = \csc(A)\sec(A) - \tan(A)$ Click here to see the solution to **1.e.**

Solution to **1.a.**

1.a. $\csc(t) - \sin(t) = \cot(t)\cos(t)$

Let's start with the left side of the identity since it involves subtraction which allows us to begin by performing the subtraction and combining the expressions being subtracted:

$$\begin{aligned}
\csc(t) - \sin(t) &= \frac{1}{\sin(t)} - \sin(t)\cdot\frac{\sin(t)}{\sin(t)} \\
&= \frac{1}{\sin(t)} - \frac{\sin^2(t)}{\sin(t)} \\
&= \frac{1 - \sin^2(t)}{\sin(t)} \\
&= \frac{\cos^2(t)}{\sin(t)} \\
&= \frac{\cos(t)}{\sin(t)}\cdot\frac{\cos(t)}{1} \\
&= \cot(t)\cos(t)
\end{aligned}$$

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Solution to **1.b.**

1.b. $\tan^2(x)\sin^2(x) = \tan^2(x) - \sin^2(x)$

Let's start with the right side of the identity since it involves subtraction that we can perform to begin our proof:

$$\begin{aligned}
\tan^2(x) - \sin^2(x) &= \frac{\sin^2(x)}{\cos^2(x)} - \sin^2(x)\cdot\frac{\cos^2(x)}{\cos^2(x)} \\
&= \frac{\sin^2(x)}{\cos^2(x)} - \frac{\sin^2(x)\cdot\cos^2(x)}{\cos^2(x)} \\
&= \frac{\sin^2(x) - \sin^2(x)\cdot\cos^2(x)}{\cos^2(x)} \\
&= \frac{\sin^2(x)\cdot\left(1 - \cos^2(x)\right)}{\cos^2(x)} \\
&= \frac{\sin^2(x)\cdot\sin^2(x)}{\cos^2(x)} \\
&= \frac{\sin^2(x)}{\cos^2(x)}\cdot\frac{\sin^2(x)}{1} \\
&= \tan^2(x)\sin^2(x)
\end{aligned}$$

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Solution to **1.c.**

1.c. $\sec(\theta) + \tan(\theta) = \dfrac{\cos(\theta)}{1 - \sin(\theta)}$

Let's start with the left side of the identity since it involves addition which allows us to begin by performing the addition and combining the expressions being added:

$$\begin{aligned}\sec(\theta) + \tan(\theta) &= \frac{1}{\cos(\theta)} + \frac{\sin(\theta)}{\cos(\theta)}\\ &= \frac{1+\sin(\theta)}{\cos(\theta)}\end{aligned}$$

At this point, I'm stuck since there's nothing obvious that can be done to $\frac{1+\sin(\theta)}{\cos(\theta)}$ to manipulate it. One good strategy to employ when you get stuck is to try working with the other side of the identity but, in this case, the other side of the identity presents a similar situation: there's nothing obvious that can be done to $\frac{\cos(\theta)}{1-\sin(\theta)}$ to manipulate it. Another strategy to try when you get stuck is to "use conjugates". The conjugate of the expression $a+b$ is the expression $a-b$, and one thing that's special about conjugates is that their product is a difference of squares: $(a+b)\cdot(a-b) = a^2 - b^2$. This comes in handy when working with sine and cosine since they're related by the Pythagorean Identity which can be used to create differences of squares. To make this more meaningful, let's consider an example: Notice that "$1+\sin(\theta)$" is in the numerator of the expression in our last step of our proof; the conjugate of that expression is "$1-\sin(\theta)$". Let's compute the product of these conjugates:

$$\begin{aligned}\big(1+\sin(\theta)\big)\cdot\big(1-\sin(\theta)\big) &= 1-\sin^2(\theta)\\ &= \cos^2(\theta)\end{aligned}$$

So, the product of these conjugates leads us to "$\cos^2(\theta)$"! Now let's try using conjugates to finish our proof; we'll re-write the first two steps:

$$\begin{aligned}\sec(\theta) + \tan(\theta) &= \frac{1}{\cos(\theta)} + \frac{\sin(\theta)}{\cos(\theta)}\\ &= \frac{1+\sin(\theta)}{\cos(\theta)}\cdot\frac{1-\sin(\theta)}{1-\sin(\theta)}\\ &= \frac{1-\sin^2(\theta)}{\cos(\theta)\cdot\big(1-\sin(\theta)\big)}\\ &= \frac{\cos^{\cancel{2}}(\theta)}{\cancel{\cos(\theta)}\cdot\big(1-\sin(\theta)\big)}\\ &= \frac{\cos(\theta)}{1-\sin(\theta)}\end{aligned}$$

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Solution to **1.d.**

1.d. $\dfrac{1}{1-\cos(x)} - \dfrac{1}{1+\cos(x)} = 2\cot(x)\csc(x)$

Let's start with the left side of the identity since it involves subtraction which allows us to begin by performing the subtraction and combining the expressions. Note that finding a common denominator will require us to employ conjugates:

$$\begin{aligned}
\frac{1}{1-\cos(x)} - \frac{1}{1+\cos(x)} &= \frac{1}{1-\cos(x)}\cdot\frac{1+\cos(x)}{1+\cos(x)} - \frac{1}{1+\cos(x)}\cdot\frac{1-\cos(x)}{1-\cos(x)} \\
&= \frac{1+\cos(x)-\left(1-\cos(x)\right)}{1-\cos^2(x)} \\
&= \frac{1+\cos(x)-1+\cos(x)}{1-\cos^2(x)} \\
&= \frac{2\cos(x)}{\sin^2(x)} \\
&= 2\cdot\frac{\cos(x)}{\sin(x)}\cdot\frac{1}{\sin(x)} \\
&= 2\cot(x)\csc(x)
\end{aligned}$$

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Solution to **1.e.**

1.e. $\cot(A) = \csc(A)\sec(A) - \tan(A)$

Let's start with the right side of the identity since it involves subtraction that we can perform to begin our proof:

$$\begin{aligned}
\csc(A)\sec(A) - \tan(A) &= \frac{1}{\sin(A)} \cdot \frac{1}{\cos(A)} - \frac{\sin(A)}{\cos(A)} \\
&= \frac{1}{\sin(A)\cos(A)} - \frac{\sin(A)}{\cos(A)} \cdot \frac{\sin(A)}{\sin(A)} \\
&= \frac{1}{\sin(A)\cos(A)} - \frac{\sin^2(A)}{\sin(A)\cos(A)} \\
&= \frac{1 - \sin^2(A)}{\sin(A)\cos(A)} \\
&= \frac{\cos^{\cancel{2}}(A)}{\sin(A)\cancel{\cos(A)}} \\
&= \frac{\cos(A)}{\sin(A)} \\
&= \cot(A)
\end{aligned}$$

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