Extra Practice for Section II: Chapter 2

1. In parts (**a**) – (**e**), you are given some info about a non-right triangle that has sides a, b, and c and angles A, B, and C oriented as shown in Figure 1. Find the length of the missing side(s) and the measure of the missing angle(s). (Unlike most of what we've done so far this quarter, it's appropriate to use a calculator for these problems in order to approximate values.)

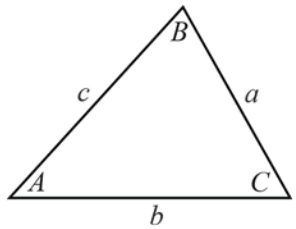

Figure 1

a. $a = 5$, $b = 6$, $c = 7$ — Click here to see the solution to **1.a.**

b. $B = 76^\circ$, $a = 8$, $c = 6$ — Click here to see the solution to **1.b.**

c. $B = 118^\circ$, $C = 37^\circ$, $a = 5$ — Click here to see the solution to **1.c.**

d. $A = 28^\circ$, $a = 7$, $c = 12$ — Click here to see the solution to **1.d.**

e. $A = 40^\circ$, $a = 11$, $c = 8$ — Click here to see the solution to **1.e.**

Solution to **1.a.**

1.a. $a = 5$, $b = 6$, $c = 7$

First, let's draw a sketch of the situation in Figure 2:

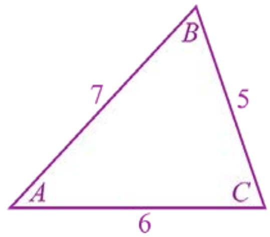

Figure 2

We aren't given any angle measures so we don't have any "angle and side opposite" pairs to work with so we cannot employ the Law of Sines, so we'll have to start with the Law of Cosines which we can use to find any of the angles but let's aim for A:

$$\begin{aligned} & 5^2 = 7^2 + 6^2 - 2\cdot 7\cdot 6\cdot\cos(A) \\ \Rightarrow\quad & 25 = 85 - 84\cdot\cos(A) \\ \Rightarrow\quad & 84\cdot\cos(A) = 60 \\ \Rightarrow\quad & \cos(A) = \tfrac{60}{84} \\ \Rightarrow\quad & A = \cos^{-1}\left(\tfrac{5}{7}\right) \\ \Rightarrow\quad & \approx 44.42^\circ \end{aligned}$$

Now that we know the measure of angle A, we have an "angle and side opposite" pair to work with so we can use the Law of Sines to find either of the remaining missing angles; let's aim for B since it's opposite a smaller side than is C. (Note that it's always better to use the Law of Sines to find the smaller angle since the inverse-sine function cannot produce angles larger than 90°.)

$$\begin{aligned} & \frac{\sin(B)}{6} = \frac{\sin(A)}{5} \\ \Rightarrow\quad & \sin(B) \approx \frac{6\cdot\sin(44.42^\circ)}{5} \\ \Rightarrow\quad & B \approx \sin^{-1}\left(\frac{6\cdot\sin(44.42^\circ)}{5}\right) \\ \Rightarrow\quad & B \approx 57.13^\circ \end{aligned}$$

Finally, we can use the fact that the sum of the angles in a triangle is 180° to find C:

$$\begin{aligned} C &= 180^\circ - A - B \\ &\approx 180^\circ - 44.42^\circ - 57.13^\circ \\ &\approx 78.45^\circ \end{aligned}$$

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Solution to **1.b.**

1.b. $B = 76^\circ$, $a = 8$, $c = 6$

First, let's draw a sketch of the situation:

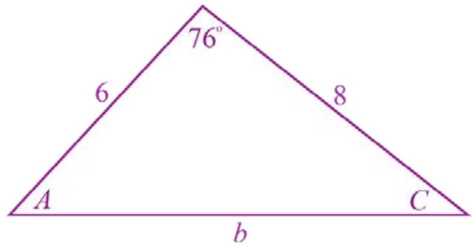

Figure 3

As with **1a**, we aren't given an "angle and side opposite" pair to work with so we cannot employ the Law of Sines but we are given the length of the two sides that form the angle $B = 76^\circ$ so we can use the Law of Cosines to find b:

$$\begin{aligned} & b^2 = 6^2 + 8^2 - 2\cdot 6\cdot 8\cdot\cos(76^\circ) \\ \Rightarrow\quad & b^2 = 100 - 96\cdot\cos(76^\circ) \\ \Rightarrow\quad & b = \sqrt{100 - 96\cdot\cos(76^\circ)} \\ \Rightarrow\quad & b \approx 8.76 \end{aligned}$$

Now that we know the length of side b, we have an "angle and side opposite" pair to work with so we can use the Law of Sines to find either of the remaining missing angles; let's aim for C since it's opposite a smaller side than is A. (Note that it's always better to use the Law of Sines to find the smaller angle since the inverse-sine function cannot produce angles larger than 90°.)

$$\begin{aligned} & \frac{\sin(C)}{6} = \frac{\sin(76^\circ)}{b} \\ \Rightarrow\quad & \sin(C) \approx \frac{6\cdot\sin(76^\circ)}{8.76} \\ \Rightarrow\quad & C \approx \sin^{-1}\left(\frac{6\cdot\sin(76^\circ)}{8.76}\right) \\ \Rightarrow\quad & C \approx 41.65^\circ \end{aligned}$$

Finally, we can use the fact that the sum of the angles in a triangle is 180° to find A:

$$\begin{aligned} A &= 180^\circ - 76^\circ - C \\ &\approx 180^\circ - 76^\circ - 41.65^\circ \\ &\approx 62.35^\circ \end{aligned}$$

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Solution to **1.c.**

1.c. $B = 118^\circ$, $C = 37^\circ$, $a = 5$

First, let's draw a sketch of the situation:

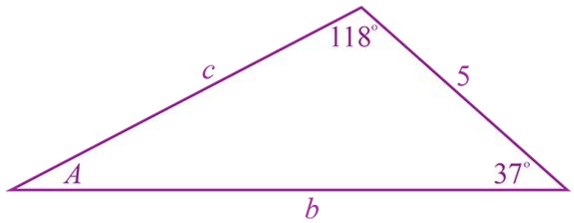

Figure 4

First, we can use the fact that the sum of the angles in a triangle is 180° to find A:

$$\begin{aligned} A &= 180^\circ - 118^\circ - 37^\circ \\ &= 25^\circ \end{aligned}$$

Now we have an "angle and side opposite" pair to work with so we can use the Law of Sines to find b and c:

$$\frac{b}{\sin(118^\circ)} = \frac{5}{\sin(A)}$$

$$\Rightarrow \qquad b = \frac{5 \cdot \sin(118^\circ)}{\sin(25^\circ)}$$

$$\Rightarrow \qquad b \approx 10.45$$

and

$$\frac{c}{\sin(37^\circ)} = \frac{5}{\sin(A)}$$

$$\Rightarrow \qquad c = \frac{5 \cdot \sin(37^\circ)}{\sin(25^\circ)}$$

$$\Rightarrow \qquad c \approx 7.12$$

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Solution to **1.d.**

1.d. $A = 28^\circ$, $a = 7$, $c = 12$

First, let's draw a sketch of the situation:

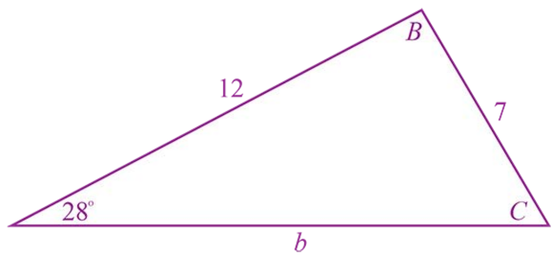

Figure 5

Since $7 < 12$, the side of length 7 units can pivot at angle B without changing any of the known info, i.e., there are two different triangles that satisfy the given information. In Figure 6, we've drawn both of these triangles, one green and one red. (Compare this situation to **1a**, **1b** , and **1c** where any attempt to pivot any of the sides would distort some of the given information so such pivoting isn't possible and there's only one possible triangle.)

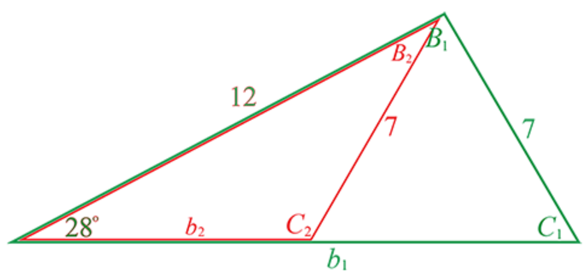

Figure 6

Now we can find the values of C_1 and C_2 using the Law of Sines. At the start, we'll represent the angle with the generic symbol "C" and wait to use subscripts until we're differentiating between the two possible values.

$$\frac{\sin(C)}{12} = \frac{\sin(28^\circ)}{7}$$

$$\Rightarrow \quad \sin(C) = \frac{12 \cdot \sin(28^\circ)}{7}$$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{12 \cdot \sin(28^\circ)}{7}\right)$$

Now we can start distinguishing between C_1 and C_2 so that we can find these two angles. The largest angle that inverse-sine can output is 90° but C_2 in the red triangle in Figure 2 is clearly greater than 90°: to find C_2 we'll need to employ the identity $\sin(\theta) = \sin(180^\circ - \theta)$:

$$C_1 = \sin^{-1}\left(\frac{12\cdot\sin(28^\circ)}{7}\right) \approx 53.59^\circ \qquad \text{or} \qquad C_2 = 180^\circ - \sin^{-1}\left(\frac{12\cdot\sin(28^\circ)}{7}\right) \approx 126.41^\circ$$

To finish, we can solve for the two possible triangles:

Possibility 1: The Green Triangle

$$C_1 \approx 53.59^\circ$$

$$\begin{aligned} B_1 &= 180^\circ - 28^\circ - C_1 \\ &\approx 180^\circ - 28^\circ - 53.59^\circ \\ &\approx 98.41^\circ \end{aligned}$$

$$\frac{b_1}{\sin(B_1)} = \frac{7}{\sin(28^\circ)}$$

$$\Rightarrow \quad b_1 = \frac{7\cdot\sin(B_1)}{\sin(28^\circ)}$$

$$\Rightarrow \quad b_1 \approx \frac{7\cdot\sin(98.41^\circ)}{\sin(28^\circ)} \approx 14.75$$

Possibility 2: The Red Triangle

$$C_2 \approx 126.41^\circ$$

$$\begin{aligned} B_2 &= 180^\circ - 28^\circ - C_2 \\ &\approx 180^\circ - 28^\circ - 126.41^\circ \\ &\approx 25.59^\circ \end{aligned}$$

$$\frac{b_2}{\sin(B_2)} = \frac{7}{\sin(28^\circ)}$$

$$\Rightarrow \quad b_2 = \frac{7\cdot\sin(B_2)}{\sin(28^\circ)}$$

$$\Rightarrow \quad b_2 \approx \frac{7\cdot\sin(25.59^\circ)}{\sin(28^\circ)} \approx 6.44$$

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Solution to **1.e.**

1.e. $A = 40^\circ$, $a = 11$, $c = 8$

First, let's draw a sketch of the situation:

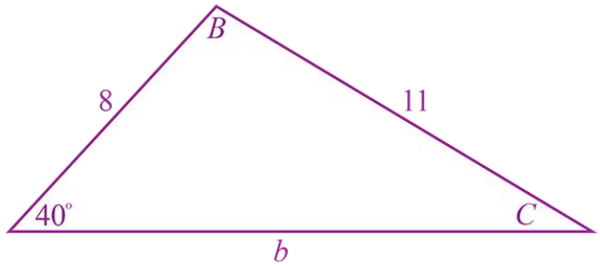

Figure 7

This is similar to the situation in **1d** but, since $11 > 8$, if we attempt to pivot the side of length 11 units at angle B, it won't line-up with side b until it's "outside" the triangle, thus making it impossible to satisfy the given requirement that $A = 40^\circ$. Thus, we cannot pivot any of the sides and there is only one possible triangle that satisfies the given information.

To solve the triangle, we can start by using the Law of Sines to find C:

$$\frac{\sin(C)}{8} = \frac{\sin(40^\circ)}{11}$$

$$\Rightarrow \quad \sin(C) = \frac{8 \cdot \sin(40^\circ)}{11}$$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{8 \cdot \sin(40^\circ)}{11}\right) \approx 27.87^\circ$$

Now we can use the fact that the sum of the angles in a triangle is 180° to find B:

$$\begin{aligned} B &= 180^\circ - 40^\circ - C \\ &\approx 180^\circ - 40^\circ - 27.87^\circ \\ &\approx 112.13^\circ \end{aligned}$$

Finally, we can use the Law of Sines to find b:

$$\frac{b}{\sin(B)} = \frac{11}{\sin(40^\circ)}$$

$$\Rightarrow \quad b = \frac{11 \cdot \sin(B)}{\sin(40^\circ)}$$

$$\Rightarrow \quad b \approx \frac{11 \cdot \sin(112.13^\circ)}{\sin(40^\circ)} \approx 15.85$$

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