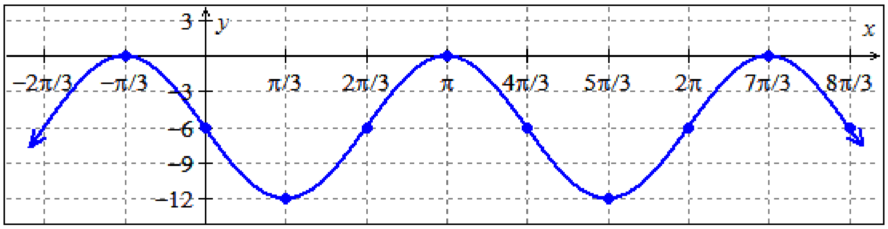
Extra Practice for Section I: Chapter 4

You should complete all of these problems **without a calculator** in order to prepare for the Midterm which is a no-calculator exam.

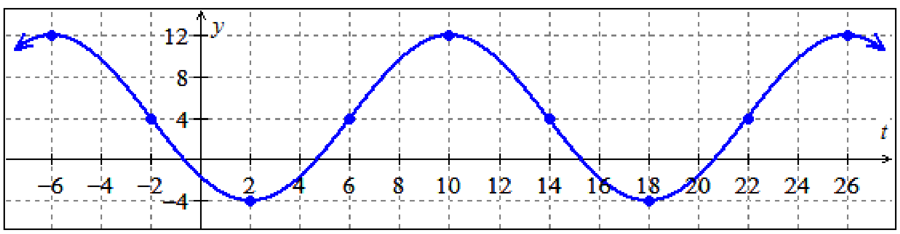
1. Find two different algebraic rules for the function $y = p(x)$ graphed below. One of your rules should involve sine and the other should involve cosine.



The graph of $y = p(x)$.

Click here to see the solution to **1.**

2. Find two different algebraic rules for the function $y = q(t)$ graphed below. One of your rules should involve sine and the other should involve cosine.



The graph of $y = q(t)$.

Click here to see the solution to **2.**

3. Draw a graph of at least two periods of the following functions. List the period, midline, and amplitude of each function. While drawing the graphs in #3, first plot the points that fall on the midline and the points where the function reaches its maximum and minimum values, and then connect these points with an appropriately curved sinusoidal wave. (Be sure to label the scale on the axes of your graph so that it has meaning.)

a. $f(t) = -5\cos(4t) + 3$ Click here to see the solution for **3.a.**

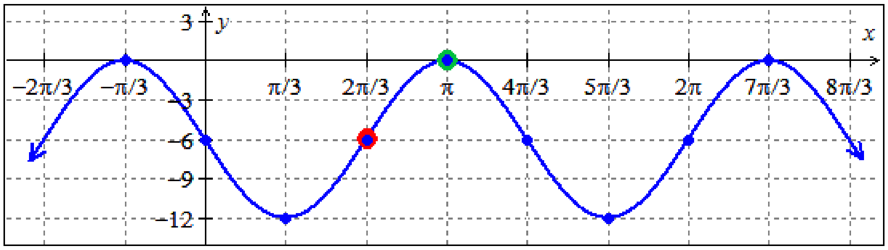
b. $g(x) = 4\sin\left(\pi\left(x - \frac{1}{4}\right)\right) - 2$ Click here to see the solution for **3.b.**

c. $G(x) = 3\cos\left(2x + \frac{\pi}{2}\right) + 4$ Click here to see the solution for **3.c.**

d. $F(t) = -2\sin\left(\frac{\pi}{3}t + \pi\right) - 1$ Click here to see the solution for **3.d.**

Solution to 1.

1. Find two different algebraic rules for the function $y = p(x)$ graphed below. One of your rules should involve sine and the other should involve cosine.



The graph of $y = p(x)$.

First let's write a rule involving sine, so our rule will have the form $p(x) = A\sin\big(\omega(x - h)\big) + k$ and we need to determine the values of A, ω, h, and k.

- The midline is the line midway between the function's maximum and minimum output values. The function's maximum output value is 0 and its minimum output value is -12. Since -6 is the average of these values, the midline is $y = -6$ so $k = -6$.

- The amplitude is the distance between the function's maximum output value, 0, and its midline $y = -6$, which is 6 units. Therefore, $|A| = 6$.

- The function completes one period between $x = \frac{2\pi}{3}$ and $x = 2\pi$. Thus, the period of the function is $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$. To find ω we need to solve $\frac{4\pi}{3} = 2\pi \cdot \frac{1}{\omega}$:

$$\begin{aligned} & \frac{4\pi}{3} = 2\pi \cdot \frac{1}{\omega} \\ \Rightarrow\quad & \omega = 2\pi \cdot \frac{1}{4\pi/3} \\ \Rightarrow\quad & \omega = 2\pi \cdot \frac{3}{4\pi} \\ \Rightarrow\quad & \omega = \frac{3}{2} \end{aligned}$$

- Near the *y*-axis, the graph of $y = \sin(x)$ is increasing and passes through its midline, so we need to look for a spot in the graph of $y = p(x)$ where it shows this behavior, and one such spot is at $x = \frac{2\pi}{3}$ (this point has been highlighted in red in the graph above) so we can so consider this graph a sine wave shifted right $\frac{2\pi}{3}$ units and use $h = \frac{2\pi}{3}$.

Therefore, an algebraic rule for the graphed function is $p(x) = 6\sin\left(\frac{3}{2}\left(x - \frac{2\pi}{3}\right)\right) - 6$.

Now we'll write a rule involving cosine.

Since we want to use cosine to construct our rule, it will have the form $p(x) = A\cos\left(\omega(x-h)\right) + k$. Since the amplitude, period, and midline aren't dependent on whether we use sine or cosine in our algebraic rule, we can use the same values for A, ω, and k that we used above. So we only need to determine an appropriate horizontal shift, h, that works for cosine. Near the y-axis, the graph of $y = \cos(x)$ is reaches its maximum value, so we need to look for a spot in the graph of $y = p(x)$ where it shows this behavior, and one such spot is at $x = \pi$ (this point has been highlighted in green in the graph above) so we can consider this graph a cosine wave shifted right π units and use $h = \pi$.

Therefore, an algebraic rule for the graphed function is $p(x) = 6\cos\left(\frac{3}{2}(x-\pi)\right) - 6$.

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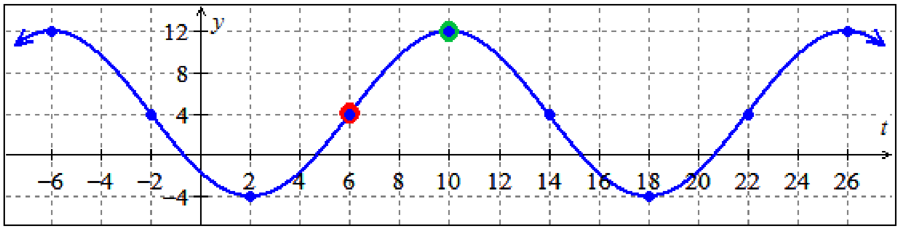
Solution to **2.**

2. Find two different algebraic rules for the function $y = q(t)$ graphed below. One of your rules should involve sine and the other should involve cosine.



The graph of $y = q(t)$.

First let's write a rule involving sine, so our rule will have the form $q(t) = A\sin\big(\omega(t-h)\big) + k$ and we need to determine the values of A, ω, h, and k.

- The midline is the line midway between the function's maximum and minimum output values. The function's maximum output value is 12 and its minimum output value is -4. Since 4 is the average of these values, the midline is $y = 4$ so $k = 4$.

- The amplitude is the distance between the function's maximum output value, 12, and its midline $y = 4$, which is 8 units. Therefore, $|A| = 8$.

- The function completes one period between $t = 6$ and $t = 22$. Thus, the period of the function is $22 - 6 = 16$. To find ω we need to solve $16 = 2\pi \cdot \frac{1}{\omega}$:

$$16 = 2\pi \cdot \frac{1}{\omega}$$
$$\Rightarrow \quad \omega = 2\pi \cdot \frac{1}{16} = \frac{\pi}{8}$$

- Near the y-axis, the graph of $y = \sin(t)$ is increasing and passes through its midline, so we need to look for a spot in the graph of $y = q(t)$ where it shows this behavior, and one such spot is at $t = 6$ (this point has been highlighted in red in the graph above) so we can consider this graph a sine wave shifted right 6 units and use $h = 6$.

- Therefore, an algebraic rule for the graphed function is $q(t) = 8\sin\left(\frac{\pi}{8}(t-6)\right) + 4$.

Now we'll write a rule involving cosine, so our rule will have the form $q(t) = A\cos\big(\omega(t-h)\big) + k$. Since the amplitude, period, and midline aren't dependent on whether we use sine or cosine in our algebraic rule, we can use the same values for A, ω, and k that we used above. So we only need to determine an appropriate horizontal shift, h, that works for cosine. Near the y-axis, the graph of $y = \cos(t)$ is reaches its maximum value, so we need to look for a spot in the graph of $y = q(t)$ where it shows this behavior, and one such spot is at $t = 10$ (this point has been highlighted in green in the graph above) so we can consider this graph a cosine wave shifted right 10 units and use $h = 10$. Therefore, an algebraic rule for the graphed function is $q(t) = 8\cos\left(\frac{\pi}{8}(t-10)\right) + 4$.

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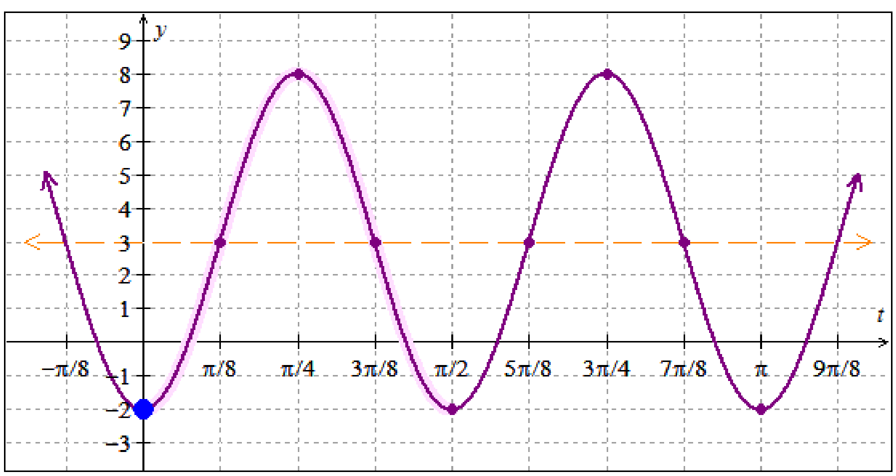
3. Draw a graph of at least two periods of the following functions. List the period, midline, and amplitude of each function. While drawing the graphs in #3, first plot the points that fall on the midline and the points where the function reaches its maximum and minimum values, and then connect these points with an appropriately curved sinusoidal wave. (Be sure to label the scale on the axes of your graph so that it has meaning.)

Solution to **3.a.**

a. $f(t) = -5\cos(4t) + 3$

- $|A| = |-5| = 5$ so the **amplitude** is 5 units. Since $A < 0$, we'll need to draw a "reflected cosine wave".
- $k = 3$ so the **midline** is $y = 3$.
- $\omega = 4$ so the **period** is $2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$ units.
- There is no horizontal shift so we'll "start" a reflected cosine wave on the y-axis and make sure it has the appropriate midline, amplitude, and period; we've highlighted the "first" period in pink. (Since this is a "reflected cosine wave", it needs to start at a minimum output value rather than at its maximum output value like $y = \cos(t)$.)



A graph of $f(t) = -5\sin(4t) + 3$.

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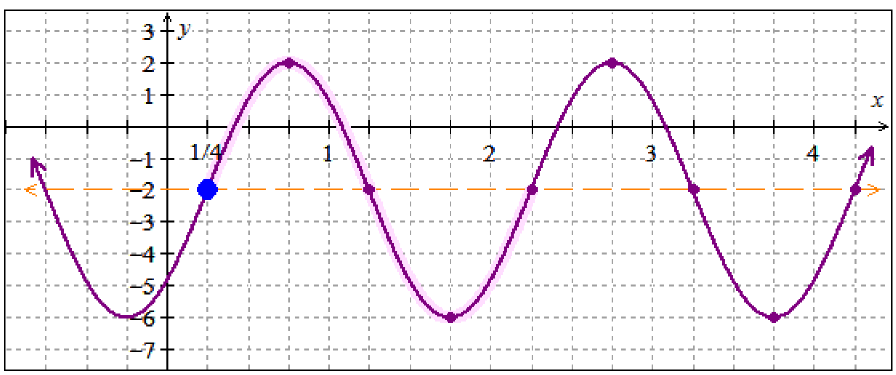
Solution to **3.b.**

b. $g(x) = 4\sin\left(\pi\left(x - \frac{1}{4}\right)\right) - 2$

- $|A| = |4| = 4$ so the **amplitude** is 4 units.
- $k = -2$ so the **midline** is $y = -2$.
- $\omega = \pi$ so the **period** is $2\pi \cdot \frac{1}{\pi} = 2$ units.
- $h = \frac{1}{4}$ so the horizontal shift is $\frac{1}{4}$ units to the right, so we'll "start" a sine wave at $x = \frac{1}{4}$ and make sure it has the appropriate midline, amplitude, and period; we've highlighted the "first" period in pink..



A graph of $g(x) = 4\sin\left(\pi\left(x - \frac{1}{4}\right)\right) - 2$.

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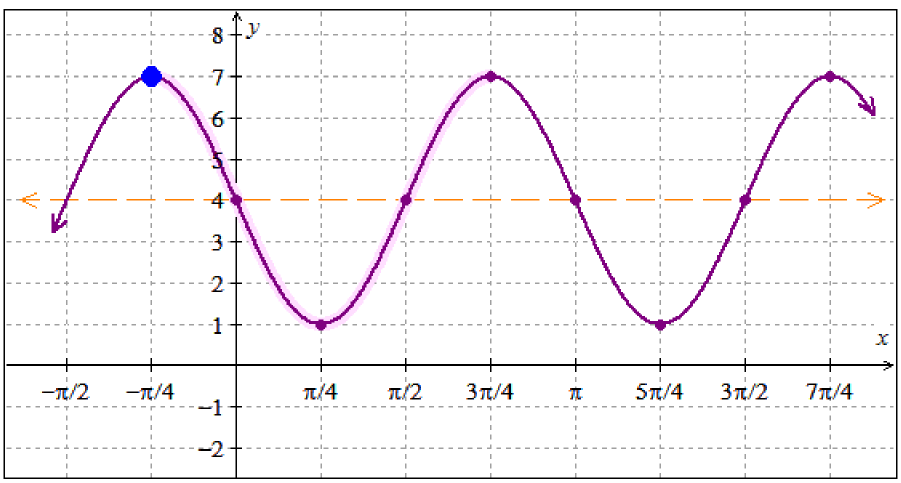
Solution to **3.c.**

c. $G(x) = 3\cos\left(2x + \frac{\pi}{2}\right) + 4$

$$= 3\cos\left(2\left(x - \left(-\frac{\pi}{4}\right)\right)\right) + 4$$

- $|A| = |3| = 3$ so the **amplitude** is 3 units.
- $k = 4$ so the **midline** is $y = 4$.
- $\omega = 2$ so the **period** is $2\pi \cdot \frac{1}{2} = \pi$ units.
- $h = -\frac{\pi}{4}$ so the horizontal shift is $\frac{\pi}{4}$ units to the left, so we'll "start" a cosine wave at $x = -\frac{\pi}{4}$ and make sure it has the appropriate midline, amplitude, and period; we've highlighted the "first" period in pink..



A graph of $G(x) = 3\cos\left(2x + \frac{\pi}{2}\right) + 4$.

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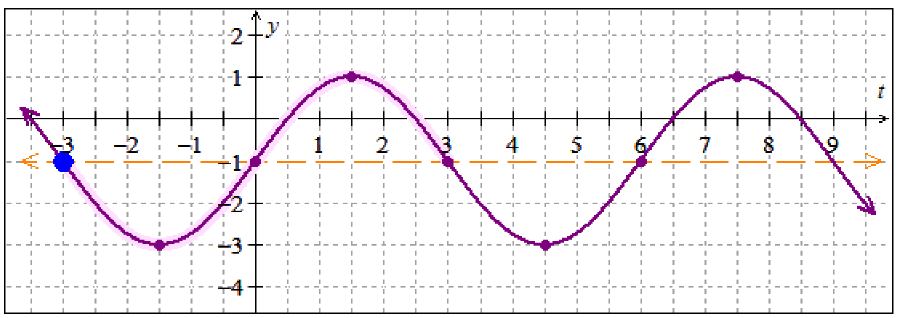
Solution to **3.d.**

d. $F(t) = -2\sin\left(\frac{\pi}{3}t + \pi\right) - 1$

$$= -2\sin\left(\frac{\pi}{3}(t - (-3))\right) - 1$$

- $|A| = |-2| = 2$ so the **amplitude** is 2 units. Since $A < 0$, we'll need to draw a "reflected sine wave".
- $k = -1$ so the **midline** is $y = -1$.
- $\omega = \frac{\pi}{3}$ so the **period** is $2\pi \cdot \frac{1}{\pi/3} = 6$ units.
- $h = -3$ so the horizontal shift is 3 units to the left, so we'll "start" a reflected sine wave at $t = -3$ and make sure it has the appropriate midline, amplitude, and period; we've highlighted the "first" period in pink. (Since this is a "reflected sine wave", it needs to travel *down* from its starting point at its midline.)



A graph of $F(t) = -2\sin\left(\frac{\pi}{3}t + \pi\right) - 1$.

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