Extra Practice for Section I: Chapter 3, Parts 1–4

You should complete all of these problems **without a calculator** in order to prepare for the Midterm which is a no-calculator exam.

1. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\cos(\theta) = \frac{\sqrt{5}}{4}$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\sin(\theta) =$ ____ since you're asked to tell me what $\sin(\theta)$ equals.)

a. $\sin(\theta)$ **b.** $\tan(\theta)$ **c.** $\sec(\theta)$ **d.** $\csc(\theta)$

Click here to see the solution to **1.**

2. Find the ***exact*** value for each of the following expressions. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. If the given expression is undefined write, "*The expression is undefined*." An example has been provided to clarify how your response should look.

ex. $\sin(0)$

sin(0)=0

Be sure to write all of <u>this</u> to communicate what $\sin(0)$ equals.

a. $\cos\left(\frac{\pi}{6}\right)$.	**b.** $\sin\left(\frac{\pi}{4}\right)$.	**c.** $\cos\left(60^{\circ}\right)$.
d. $\sin\left(-\frac{5\pi}{3}\right)$.	**e.** $\cos\left(\frac{7\pi}{6}\right)$.	**f.** $\sin\left(240^{\circ}\right)$.
g. $\sin\left(\frac{17\pi}{6}\right)$.	**h.** $\sec(2\pi)$.	**i.** $\cos\left(135^{\circ}\right)$.
j. $\tan\left(\frac{4\pi}{3}\right)$.	**k.** $\csc(\pi)$.	**l.** $\sec\left(\frac{3\pi}{4}\right)$.

Click here to see the solution to **2.**

3. The point P in Figure 1 is specified by $\frac{5\pi}{6}$ on the circumference of a circle with a radius of 12 units. Use the *sine and cosine function to find the* ***exact*** *coordinates of* P.

[**Be sure to *show* your use of sine and cosine.**]

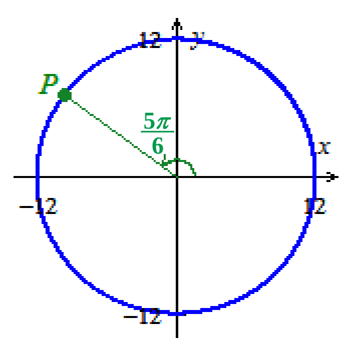

Figure 1

Click here to see the solution to **3.**

Solution to **1.**

1. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\cos(\theta) = \frac{\sqrt{5}}{4}$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\sin(\theta) =$ ____ since you're asked to tell me what $\sin(\theta)$ equals.)

a. $\sin(\theta)$

To find $\sin(\theta)$, we can use the Pythagorean Identity: $\sin^2(\theta) + \cos^2(\theta) = 1$:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\Rightarrow \sin^2(\theta) + \left(\frac{\sqrt{5}}{4}\right)^2 = 1$$

$$\Rightarrow \sin^2(\theta) + \frac{5}{16} = 1$$

$$\Rightarrow \sin^2(\theta) = 1 - \frac{5}{16}$$

$$\Rightarrow \sin(\theta) = -\sqrt{\frac{11}{16}}$$

(since $\frac{3\pi}{2} < \theta < 2\pi$, $\sin(\theta) < 0$ so we take the negative square root)

$$\Rightarrow \sin(\theta) = -\frac{\sqrt{11}}{4}$$

b. $\tan(\theta)$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{-\sqrt{11}/4}{\sqrt{5}/4}$$

$$= \frac{-\sqrt{11}}{\sqrt{5}} = \frac{-\sqrt{55}}{5}$$

(you aren't required to rationalize the denominator)

c. $\sec(\theta)$

$$\begin{aligned}\sec(\theta) &= \frac{1}{\cos(\theta)} \\ &= \frac{1}{\sqrt{5}/4} \\ &= \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}\end{aligned}$$

(you aren't required to rationalize the denominator)

d. $\csc(\theta)$

$$\begin{aligned}\csc(\theta) &= \frac{1}{\sin(\theta)} \\ &= \frac{1}{-\sqrt{11}/4} \\ &= -\frac{4}{\sqrt{11}} = -\frac{4\sqrt{11}}{11}\end{aligned}$$

(you aren't required to rationalize the denominator)

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Solution to **2.**

2. Find the ***exact*** value for each of the following expressions. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. If the given expression is undefined write, "*The expression is undefined*." An example has been provided to clarify how your response should look.

ex. $\sin(0)$

$\sin(0) = 0$

At a minimum, write all of <u>this</u> to communicate what $\sin(0)$ *equals.*

a. $\cos\left(\frac{\pi}{6}\right)$.

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

b. $\sin\left(\frac{\pi}{4}\right)$.

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

c. $\cos\left(60^\circ\right)$.

$\cos\left(60^\circ\right) = \frac{1}{2}$

d. $\sin\left(-\frac{5\pi}{3}\right)$.

$$\begin{aligned}\sin\left(-\frac{5\pi}{3}\right) &= \sin\left(\frac{\pi}{3}\right)\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

e. $\cos\left(\frac{7\pi}{6}\right)$.

$$\begin{aligned}\cos\left(\frac{7\pi}{6}\right) &= -\cos\left(\frac{\pi}{6}\right)\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

f. $\sin\left(240^\circ\right)$.

$$\begin{aligned}\sin\left(240^\circ\right) &= -\sin\left(60^\circ\right)\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

g. $\sin\left(\frac{17\pi}{6}\right)$.

$$\begin{aligned}\sin\left(\frac{17\pi}{6}\right) &= \sin\left(\frac{\pi}{6}\right)\\ &= \frac{1}{2}\end{aligned}$$

h. $\sec(2\pi)$.

$$\begin{aligned}\sec(2\pi) &= \frac{1}{\cos(2\pi)}\\ &= \frac{1}{1}\\ &= 1\end{aligned}$$

i. $\cos\left(135^\circ\right)$.

$$\begin{aligned}\cos\left(135^\circ\right) &= -\cos\left(45^\circ\right)\\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

j. $\tan\left(\frac{4\pi}{3}\right)$.

$$\begin{aligned}\tan\left(\frac{4\pi}{3}\right) &= \frac{\sin\left(\frac{4\pi}{3}\right)}{\cos\left(\frac{4\pi}{3}\right)}\\ &= \frac{-\sqrt{3}/2}{-1/2}\\ &= \sqrt{3}\end{aligned}$$

k. $\csc(\pi)$.

$\csc(\pi) = \frac{1}{\sin(\pi)}$

Since $\sin(\pi) = 0$,
$\csc(\pi)$ is undefined.

l. $\sec\left(\frac{3\pi}{4}\right)$.

$$\begin{aligned}\sec\left(\frac{3\pi}{4}\right) &= \frac{1}{\cos\left(\frac{3\pi}{4}\right)}\\ &= \frac{1}{-\sqrt{2}/2}\\ &= -\frac{2}{\sqrt{2}}\\ &= -\sqrt{2}\end{aligned}$$

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Solution to **3.**

3. The point P in Figure 1 is specified by $\frac{5\pi}{6}$ on the circumference of a circle with a radius of 12 units. Use the *sine and cosine function to find the* ***exact*** *coordinates of* P.
[**Be sure to *show* your use of sine and cosine**.]

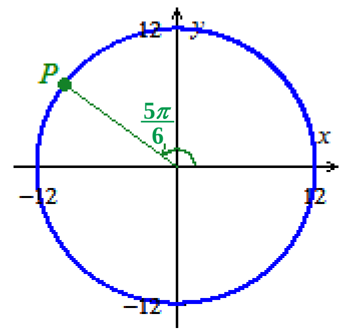

Figure 1

Coordinates of P:

$$\left(12\cos\left(\frac{5\pi}{6}\right),\ 12\sin\left(\frac{5\pi}{6}\right)\right) = \left(12\cdot\left(-\frac{\sqrt{3}}{2}\right),\ 12\cdot\left(\frac{1}{2}\right)\right)$$
$$= \left(-6\sqrt{3},\ 6\right)$$

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