Write your answers on a separate sheet of paper.

The Law of Sines
\[ \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \]

1. Choose the appropriate Law of Sines equality to solve the triangles for the requested values. Be sure to include a drawing of the triangle.
   a. \( A = 64^\circ, B = 98^\circ, a = 29 \) for \( b \).
   b. \( A = 50^\circ, B = 66^\circ, c = 8 \) for \( a \).

2. Find an angle on the interval \( 0^\circ \leq \theta \leq 180^\circ \) that has the same trig function value as the given angle. If such an angle does not exist, explain why.
   a. \( \sin(37.4^\circ) \)
   b. \( \cos(15.3^\circ) \)

Definition of inverse trig functions.

3. Complete the table. The first row is done for you.

<table>
<thead>
<tr>
<th>Inverse Expression</th>
<th>Trig Equation</th>
<th>Angle Interval</th>
<th>Every-Day English</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin^{-1}(z) )</td>
<td>( \sin^{-1}(z) = \theta ) so ( \sin(\theta) = z )</td>
<td>(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2})</td>
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4. Use the given number line for the following.

a. Plot the given values on a number line.
   \[ \frac{3\pi}{2}, 0, 2\pi, -\frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4} \]

b. Which angles are on the interval \( -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \)?

c. Which angles are on the interval \( 0 \leq \theta \leq \pi \)?

d. T or F \( \sin^{-1}(\sin(\frac{2\pi}{4})) = \frac{2\pi}{4} \). Briefly explain your choice.

e. T or F \( \sin^{-1}(\sin(-\frac{\pi}{4})) = -\frac{\pi}{4} \). Briefly explain your choice.
Coordinates that are not on the unit circle.

When the input of the inverse function is not from the coordinates on the unit circle, you need to use either:

Option 1: \( x^2 + y^2 = r^2 \) and the fundamental identities

Or

Option 2: The pythagorean identities and the fundamental identities.

Example:

Find the exact value of \( \csc(\tan^{-1}(-2)) \)

**Option 1**

\[
\begin{align*}
\theta &= \tan^{-1}(-2) \\
\tan(\theta) &= -\frac{2}{1} = -2 \\
1^2 + (-2)^2 &= r^2 \\
\sqrt{5} &= r \text{ (positive because it's a radius)} \\
\csc(\theta) &= \frac{r}{y} = \frac{\sqrt{5}}{2} \\
\csc(\tan^{-1}(-2)) &= -\frac{\sqrt{5}}{2}
\end{align*}
\]

**Option 2**

\[
\begin{align*}
\theta &= \tan^{-1}(-2) \\
\tan(\theta) &= -2 \text{ so } \cot(\theta) = -\frac{1}{2} \text{ (QIV)} \\
\cot^2(\theta) + 1 &= \csc^2(\theta) \\
\left(-\frac{1}{2}\right)^2 + 1 &= \csc^2(\theta) \\
\frac{5}{4} &= \csc^2(\theta) \\
-\frac{\sqrt{5}}{2} &= \csc(\theta) \text{ (negative because in QIV)} \\
\csc(\tan^{-1}(-2)) &= -\frac{\sqrt{5}}{2}
\end{align*}
\]

5. Find the exact value of \( \cos(\sin^{-1}(\frac{\sqrt{5}}{3})) \)
Answers

1. 
   a. \[
   \frac{\sin(64)}{29} = \frac{\sin(98)}{b}
   \]
   \[b \approx 31.95\]
   b. \[C = 64^\circ \text{ and } \frac{\sin(64)}{8} = \frac{\sin(50)}{a}\]
   \[a \approx 6.82\]

2. 
   a. \[142.6^\circ\]
   b. There is no such angle because the interval \(0^\circ \leq \theta \leq 180^\circ\) covers QI and QII only
      and cosine values are positive in QI, but negative in QII.

3. 
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4. 
   a. 

   b. \[-\frac{\pi}{4} \text{ and } 0\]
   c. \[0 \text{ and } \frac{3\pi}{4}\]
   d. F. \[-\frac{\pi}{4}\] is not on the interval \[-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\] so \(\sin^{-1}(\sin(\frac{\pi}{4})) \neq \frac{\pi}{4}\). Instead,
      \[\sin^{-1}(\sin(\frac{\pi}{4})) = -\frac{\pi}{4}\]. Try it on your calculator!
   e. T. \[-\frac{\pi}{4}\] is on the interval \[-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\] so \(\sin^{-1}(\sin(-\frac{\pi}{4})) = -\frac{\pi}{4}\).

5. \[\cos(\sin^{-1}(\frac{\sqrt{2}}{2})) = \frac{\sqrt{2}}{2}\].