5.9 What is wrong? Explain what is wrong in each of the following scenarios.

(a) If you toss a fair coin three times and a head appears each time, then the next toss is more likely to be a tail than a head.

A “fair” coin has by definition a 50% chance of a heads or tails outcome on a single toss. On the next toss, the probability does not change. Thus, as we change the sample space

\[
\begin{align*}
n = 1: & \{ \text{sample space } | \ H, T \} \\
n = 2: & \{ \text{sample space } | \ HH, HT, TH, TT \} \\
n = 3: & \{ \text{sample space } | \ HHH, HHT, HTH, THH, HTT, THT, TTH, TTT \}
\end{align*}
\]

the probability of getting a single head or tail outcome does not change, regardless of the space you find yourself in.

To answer the question directly \( P(T \mid H \text{ and } H \text{ and } H) = 0.5 \) because the outcome of coin tosses follow the independent events rule.

Why would someone believe \( P(T \mid H \text{ and } H \text{ and } H) > 0.5? \) Because we understand that \( P(T) = 0.5 \) is a long run value \( P(T) = \frac{\text{number of tails}}{\text{number of tosses}} \approx 0.5 \) the longer the number of tosses. Thus, since three heads appeared in a row you want a tail to appear to “even things” out to get closer to the 50% mark. This of course is a complete fallacy. Why?

1. If a run of three heads is possible then so is a run of three tails at some future time. Keep in mind the 50% is a long run value, not a short run value.
2. Also, suppose the first 100 throws are all heads, and the next 100,000 throws are divided evenly, 50,000 heads, and 50,000 tails; what is \( \hat{p} \)? \( \hat{p} = \frac{50,100}{100,000} \approx 0.5005 \). So as you can see we don’t have to make up the run of 100 heads to get close to 50/50 in the long run.

(b) If you toss a fair coin three times and a head appears each time, then the next toss is more likely to be a head than a tail.

Same reasoning as above; the probability does not change.
(c) \( \hat{p} \) is one of the parameters for a binomial distribution.

The symbol \( \hat{p} \) is a statistic not a parameter. The statistic \( \hat{p} \) is an estimate of the parameter \( p \).

5.11 Should you use the binomial distribution? In each situation below, is it reasonable to use a binomial distribution for the random variable \( X \)? Give reasons for your answer in each case. If a binomial distribution applies, give the values of \( n \) and \( p \).

(a) A poll of 200 college students asks whether or not you are usually irritable in the morning. \( X \) is the number who reply that they are usually irritable in the morning.

- There is a fixed sample size.
- The response can be categorized into success or failure, where success is “a college student is irritable in the morning.”
- \( X \) counts number of people out of the 200 who are irritable.
- We can assume that at the instance that the sample is taken, the proportion of college students that are irritable in the morning is a fixed value; \( p \) is fixed during sampling.
- Our sample size is 200 and the population size is in the millions (all college students), thus we have independence.

Thus the situation seems to fit a binomial distribution model.

(b) You toss a fair coin until a head appears. \( X \) is the count of the number of tosses that you make.

Not binomial, since the sample size is not fixed: “You toss a fair coin until a head appears.”

(c) Most calls made at random by sample surveys don't succeed in talking with a live person. Of calls to New York City, only 1/12 succeed. A survey calls 500 randomly selected numbers in New York City. \( X \) is the number that reach a live person.

- The sample size is fixed at 500.
- Our sample size is 500 and the population size is in the millions (all adults in New York City), thus we have independence.
- The response can be categorized into success or failure, where success is “a live person is reached.”
- We can assume that at the instance that the sample is taken, the proportion of college students that are irritable in the morning is a fixed value; \( p = 1/12 \) is fixed during sampling.
- \( X \) counts number of people out of the 500 who we reach via phone.

Thus the situation seems to fit a binomial distribution model.
5.13. Typographic errors. Typographic errors in a text are either non-word errors (as when “the” is typed as “the”) or word errors that result in a real but incorrect word. Spell-checking software will catch non-word errors but not word errors. Human proofreaders catch 70% of word errors. You ask a fellow student to proofread an essay in which you have deliberately made 10 word errors.

(a) If the student matches the usual 70% rate, what is the distribution of the number of errors caught? What is the distribution of the number of errors missed?

...what is the distribution?

We need to ask ourselves if this situation matches a binomial distribution. There are 10 words that are classified as word errors. During the reading when the proofreader gets to this word they will either catch the mistake or not. There is then a fixed number of trials 10; there are ten occasions in which the proofreader can make the correct decision. We will assume that the 70% success rate stays constant throughout. And we will also assume independence; if a reader misses a misspelled word the chances of correctly identifying a mistake next time is still 70%. The binomial model seems to be a good one for this case.

Thus the random variable \( X \), counts the number of times a person correctly identifies a mistake.

The values this random variable can take is \( \{X \mid 0, 1, 2, 3, \ldots, 10\} \).

Number of errors caught: \( p = 0.7, n = 10 \).

Number of errors missed; \( p = 0.3, n = 10 \). Let \( Y \) count the number of misses out of 10. \( \{Y|0, 1, 2, 3, \ldots, 10\} \)

(b) Missing 4 or more out of 10 errors seems a poor performance. What is the probability that a proofreader who catches 70% of word errors misses 4 or more out of 10?

I will use a computer program to calculate the possibilities. Thus the random variable \( Y \), counts the number of times a person misses a mistake.

I want \( P(Y \geq 4) \) that is I want to calculate the probabilities for the highlighted counts: \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \) where \( n = 10, p = 0.3 \).

\[
P(Y \geq 4) = 0.3504
\]

Using Excel: \( P(Y \geq 4) = 1 - P(Y \leq 3) \)

= 1 - binomdist(4, 10, 0.3, true)

Using TI83/84 -

\[
P(Y \geq 4) = 1 - P(Y \leq 3)
\]

= 1 - binomcdf(10, 0.3, 3)
5.15 **Typographic errors.** Return to the proofreading setting of Exercise 5.13.

(a) What is the mean number of errors caught? What is the mean number of errors missed? You see that these two means must add to 10, the total number of errors.

*The mean number of errors caught is $0.7(10) = 7$. We would expect to catch 7 errors out of 10 word errors if our probability of success is 70%.*

*The mean number of errors missed is the exact opposite 3, or $0.3(10) = 3$.*

(b) What is the standard deviation $\sigma$ of the number of errors caught?

I will use the formula $\sigma = \sqrt{np(1-p)}$

$\sigma = \sqrt{10(0.7)(0.3)} \approx 1.45$ errors

(c) Suppose the proof reader catches 90% of word errors, so that $p = 0.9$. What is $\sigma$ in this case? What is $\sigma$ if $p = 0.99$? What happens to the standard deviation of a binomial distribution as the probability of a success gets close to 1?

**Case 1:** $p = 0.9$

$\sigma = \sqrt{10(0.9)(0.1)} = 0.9487$

**Case 2:** $p = 0.99$

$\sigma = \sqrt{10(0.99)(0.01)} = 0.3146$

In general our function is $\sigma = \sqrt{10p(1-p)}$

The examples show that as $p$ gets closer to 1, the standard deviation, which measures the variation of $\hat{p}$, is reduced. This makes sense since the closer $p$ gets to 1, the more certain you are of success occurring. If you are more certain success will occur, that means most observations will be success, thus, you will hardly see failure. Therefore less variation.

When we graph this function $\sigma = \sqrt{10p(1-p)}$ as a function of $p$, we can see as $p$ approaches 1 or 0, the standard deviation of $\hat{p}$, $\sigma$, goes to zero.
5.27 A test for ESP. In a test for ESP (extrasensory perception), the experimenter looks at cards that are hidden from the subject. Each card contains either a star, a circle, a wave, or a square. As the experimenter looks at each of 20 cards in turn, the subject names the shape on the card.

(a) If a subject simply guesses the shape on each card, what is the probability of a successful guess on a single card? Because the cards are independent, the count of successes in 20 cards has a binomial distribution.

There are four card types, thus if one is guessing the probability of success is \( p = \frac{1}{4} \).

(b) What is the probability that a subject correctly guesses at least 10 of the 20 shapes?

Let the binomial random variable \( X \) count the number of correct guesses.

\[
P(X = 10) = 0.009922275
\]

Excel:

\[
P(X = 10) = \text{binomdist}(10, 20, 0.25, \text{false})
\]

TI83/84:

\[
P(X = 10) = \text{binompdf}(20, 0.25, 10)
\]

(c) In many repetitions of this experiment with a subject who is guessing, how many cards will the subject guess correctly on the average? What is the standard deviation of the number of correct guesses?

\[
\mu_{\text{count}} = 20(0.25) = 5 \quad \sigma_{\text{count}} = \sqrt{20(0.25)(0.75)} \approx 1.9365
\]

(d) A standard ESP deck actually contains 25 cards. There are five different shapes, each of which appears on 5 cards. The subject knows that the deck has this makeup. Is a binomial model still appropriate for the count of correct guesses in one pass through this deck? If so, what are \( n \) and \( p \)? If not, why not?

I am assuming that the person is going through the deck of cards by pulling a card out, asking the person to guess, then placing that card away from the deck. The choose another card from the deck of 19 remaining cards.

In the situation described above the requirement of independence is not met, and \( p \) is not fixed from observation to observation. Thus, the situation is not binomial.