1. A person who weighs 670 newtons steps onto a spring scale in the bathroom, and the spring compresses by 0.79 cm. (a) What is the spring constant? (b) What is the weight of another person who compresses the spring by 0.34 cm?

Solution

(a) By definition, the spring constant is the proportionality constant in the formula for an ideal spring: $F = kx$, where $F$ is the force required to produce the extension and/or compression of $x$ meters in the spring’s length. In this case,

$$(670) = (k)(0.0079) \implies 84810$$

(b) Since the spring is assumed to be ideal, this $k$-value works for any level of force. Thus,

$$F = (84810)(0.0034) \implies F = 288.35$$

2. A spring stretches by 0.018 meters when a 2.8-kilogram object is suspended from its end. How much mass should be attached to this spring so that its frequency of vibration is 3.0 hertz?

Solution

The frequency of vibration for a spring is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

We want $m$ for a given $f$. We need $k$ to do the calculation. This we can get from the first sentence. The weight of the object stretches the spring according to $F = kx$. Thus,

$$(2.8)(9.80) = (k)(0.018) \implies k = 1524.4$$

Now we can use the frequency formula:

$$(3.0) = \frac{1}{2\pi} \sqrt{\frac{1524.4}{m}} \implies m = 4.2904$$

3. A rifle fires a 21.0-gram pellet straight upward, because the pellet rests on a compressed spring that is released when the trigger is pulled. The spring has a negligible mass and is compressed by 9.10 cm from its unstrained length. The pellet rises to a maximum height of 6.10 meters above its position on the compressed spring. Ignoring air resistance, determine the spring constant.

Solution

In this case, the elastic potential energy stored in the spring of the rifle is transferred into the kinetic energy of the bullet. Then as the bullet rises, this kinetic energy is transferred into gravitational potential energy. At the point of maximum height, all the elastic potential energy has been converted into gravitational potential energy. Since we know the mass and height of the pellet, we can determine the energy level of the system:

$$PE_{grav} = mgh = (0.0210)(9.80)(6.10) = 1.2554$$

This energy originally came from the elastic potential energy which is given by $PE_{elas} = \frac{1}{2}kx^2$. Thus,

$$(1.2554) = \frac{1}{2}(k)(0.0910)^2 \implies k = 303.20$$
4. A 0.200-meter uniform bar has a mass of 0.750 kilogram and is released from rest in the vertical position, as Figure 1 indicates. The spring is initially unstrained and has a spring constant of $k = 25.0 \text{ N/m}$. Find the tangential speed with which end A strikes the horizontal surface.

![Figure 1: Problem 10.42](image)

**Solution**

In this problem it seems appropriate to use energy to get to the answer. There are three types involved:

- Gravitational potential energy (due to the height of the bar's center of mass)
- Elastic potential energy (due to the stretched spring)
- Rotational kinetic energy (due to the swing of the bar about the pivot point)

At the beginning of the problem these have the following values:

$$PE_{\text{grav}} = mgh = (0.750)(9.80)(0.100) = 0.73500$$
$$PE_{\text{elas}} = \frac{1}{2}kx^2 = \frac{1}{2}(25.0)(0)^2 = 0$$
$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(I)(0)^2 = 0$$

So the only energy is in the gravitational potential energy. At the end this energy will be converted into one of the other two types. There are two intermediate calculations to get out of the way. First, what is $I$, the moment of inertia of the rod? Since this is a simple rod rotating about one of its ends, its moment of inertia is

$$I = \frac{1}{4}ml^2 = \frac{1}{4}(0.750)(0.200)^2 = 0.01000$$

The second intermediate calculation involves the stretched length of the spring. Notice in the diagram that when the spring is stretched out completely that it forms the hypotenuse of a right triangle. The height of this triangle is 0.100 meters and its length is 0.200 meters. This means that the length of the hypotenuse is

$$h = \sqrt{(0.100)^2 + (0.200)^2} = 0.22361$$

But remember that the unstrained length of the spring is 0.100 meters, so the amount which it is stretched is

$$x = 0.22361 - 0.100 = 0.12361$$

Now we have enough to complete the last phase of the problem. At the end of the problem, the energy components are:

$$PE_{\text{grav}} = mgh = (0.750)(9.80)(0) = 0$$
$$PE_{\text{elas}} = \frac{1}{2}kx^2 = \frac{1}{2}(25.0)(0.12361)^2 = 0.19098$$
$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.01000)(\omega)^2 = (0.00500)(\omega)^2$$

2.09 m/s
The conservation of energy allows us to equate the before and after energy levels. Thus,
\[ 0.73500 = 0.19098 + (0.00500)(\omega)^2 \implies \omega = 10.431 \]
This is the angular speed of the rod. In order to calculate the tangential speed we use \( v = r\omega \) where \( r \) is 0.200 meters. Therefore:
\[ v = (0.200)(10.431) = 2.0862 \]

5. A simple pendulum is swinging back and forth through a small angle, its motion repeating every 1.25 seconds. How much longer should the pendulum be made in order to increase its period by 0.20 seconds?

Solution

The frequency is related to the period via \( f = 1/T \). And the frequency of a pendulum is related to is length according to
\[
 f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}
\]
Where \( g \) is the acceleration due to gravity. Thus,
\[
 \frac{1}{1.25} = \frac{1}{2\pi} \sqrt{\frac{9.80}{L}} \implies L = 0.38787
\]
In order to increase the period (or decrease the speed) we need to lengthen the pendulum. If we want the period to be 1.45 seconds, then:
\[
 \frac{1}{1.45} = \frac{1}{2\pi} \sqrt{\frac{9.80}{L}} \implies L = 0.52192
\]
The length of increase is the difference between these two lengths:
\[
 \Delta l = 0.52192 - 0.38787 = 0.13405
\]

6. One end of a piano wire is wrapped around a cylindrical tuning peg and the other end is fixed in place. The tuning peg is turned so as to stretch the wire. The piano wire is made from steel \( (Y = 2.0 \times 10^{11} \text{ N/m}^2) \). It has a radius of 0.80 mm, and an unstrained length of 0.76 meters. The radius of the tuning peg is 1.8 mm. Initially, there is no tension in the wire. Find the tension in the wire when the tuning peg is turned through two revolutions.

Solution

Since the wire is being stretched, we are dealing with longitudinal stress. Another clue is the fact that we are given Young’s modulus in the statement of the problem. The main equation to use is
\[
 F = Y \frac{\Delta L}{L_0}
\]
We are asked for the tension in the wire which is \( F \). The cross-sectional area can be obtained from the radius of the wire using \( A = \pi r^2 \). Thus,
\[
 A = \pi (0.80 \times 10^{-3})^2 = 2.0106 \times 10^{-6}
\]
We are given \( Y \) explicitly and the initial length \( L_0 \) is 0.76 meters. The change in length is due to the turning of the peg. Since the wire is made to twist twice around peg, the change in length is twice the circumference of the peg \( (C = 2\pi r) \). Thus,
\[
 \Delta L = 2C = (2)(2\pi)(1.8 \times 10^{-3}) = 2.2619 \times 10^{-2}
\]
We now have all the numbers we need. Plugging them in, we get:
\[
 \frac{(F)}{(2.0106 \times 10^{-6})} = (2.0 \times 10^{11}) \frac{2.2619 \times 10^{-2}}{0.76} \implies F = 11968
\]
7. An 86.0-kilogram climber is scaling the vertical wall of a mountain. His safety rope is made of nylon that, when stretched, behaves like a spring with a spring constant of 1200 N/m. He accidentally slips and falls freely for 0.750 meters before the rope runs out of slack. How much is the rope stretched when it breaks his fall and momentarily brings him to rest?

Solution
As the climber is falling his gravitational potential energy is being converted into the kinetic energy of his fall. The total amount of gravitational potential energy converted is

\[ PE_{\text{grav}} = mgh = (86.0)(9.80)(0.750) = 632.10 \]

This is the climber’s kinetic energy once the rope begins to stretch. At this point his kinetic energy is then converted into the elastic potential energy (\( PE_{\text{elas}} = \frac{1}{2}kx^2 \)) in the rope. However, he is still falling, so there is more gravitational potential energy to take into account (\( PE_{\text{grav}} = mgh \)). In other words, at the moment the rope begins to stretch, the types of energy are:

\[ KE = 632.10 \]
\[ PE_{\text{grav}} = mgh = (86.0)(9.80)(x) = (842.80)(x) \]
\[ PE_{\text{elas}} = \frac{1}{2}kx^2 = \frac{1}{2}(1200)(0)^2 = 0 \]

At the bottom of the stretch, the climber is momentarily at rest (no kinetic energy), and he has exhausted his gravitational potential energy, but now the spring is stretched. Now the types of energy are:

\[ KE = 0 \]
\[ PE_{\text{grav}} = mgh = (86.0)(9.80)(0) = 0 \]
\[ PE_{\text{elas}} = \frac{1}{2}kx^2 = \frac{1}{2}(1200)(x)^2 = (600)(x)^2 \]

Conservation of energy says that these two totals are the same. Thus,

\[ (632.10) + (842.80)(x) = (600)(x)^2 \]

This is a quadratic equation in \( x \). Rewriting this in standard form yields:

\[ -600x^2 + 842.80x + 632.10 = 0 \]

And applying the quadratic formula yields:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-(842.80) \pm \sqrt{(842.80)^2 - 4(-600)(632.10)}}{2(-600)} \]
\[ = \frac{-842.80 \pm 1492.4}{-1200} \]
\[ = 0.54133 \text{ or } 1.9460 \]

The negative solution corresponds to an upward stretch—we want the positive solution.

8. The front spring of a car’s suspension system has a spring constant of \( 1.50 \times 10^6 \) N/m and supports a mass of 215 kilograms. The wheel has a radius of 0.400 meters. The car is traveling on a bumpy road, on which the distance between the bumps is equal to the circumference of the wheel. Due to resonance, the wheel starts to vibrate strongly when the car is traveling at a certain minimum linear speed. What is this speed?

Solution
Resonance will occur when the driving (no pun intended) frequency matches the natural frequency of the system...

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{1.50 \times 10^6}{215}} = 13.294 \]

What this means is that the bumps in the road have to be hit at just this frequency—just over 13 bumps a second. Now, the distance between these bumps is equal to the circumference of the wheel, which has a radius of 0.400 meters. This distance is

\[ C = 2\pi r = (2\pi)(0.400) = 2.5133 \]

So, if we multiply this bump-to-bump distance by the number of bumps per second we get the linear speed required to create resonance:

\[ v = (2.5133)(13.294) = 33.412 \]

9. A 10.0-gram bullet is fired horizontally into a 2.50-kilogram wooden block attached to one end of a massless, horizontal spring \((k = 845 \text{ N/m})\). The other end of the spring is fixed in place, and the spring is unstrained initially. The block rests on a horizontal, frictionless surface. The bullet strikes the block perpendicularly and quickly comes to a halt within it. As a result of this completely inelastic collision, the spring is compressed along its axis and causes the block/bullet to oscillate with an amplitude of 0.200 meters. What is the speed of the bullet?

**Solution**

When the bullet stikes the block, the combination results in a certain velocity based on

\[ u = v_2 \frac{m_2}{m_1 + m_2} \]

Which is really just the conservation of momentum. This resulting speed is what sets the spring in motion. In fact, this speed is the maximum speed—the speed the oscillation has when it is at the equilibrium point. We know that

\[ v_{max} = A\omega \]

We are given the amplitude, but not \(\omega\). However, it is related to the natural frequency by \(\omega = 2\pi f\) and \(f = (1/2\pi)\sqrt{k/m}\), so

\[ \omega = \sqrt{\frac{k}{m}} \]

The mass of the oscillating system is \(m_1 + m_2\), so

\[ m = 0.010 + 2.50 = 2.510 \]

And we are given \(k\), so

\[ \omega = \sqrt{\frac{845}{2.510}} = 18.348 \]

Which is the natural angular frequency of the system. Therefore the maximum velocity is

\[ v_{max} = (0.200)(18.348) = 3.6696 \]

Which has to be the final velocity after the collision, \(u\). Therefore:

\[ (3.6696) = (v_2)\frac{0.010}{2.510} \implies v_2 = 921.08 \]

10. Suppose we have two complex amplitudes,

\[ a = 2.0 \exp(i\pi/2) \]
\[ b = 3.0 \exp(i\pi/3) \]
Calculate the magnitude of $a + b$.

**Solution**

Complex numbers are easy to multiply and divide when in polar form, e.g., $z = r \exp(i \theta)$. They are easy to add and subtract when in rectangular form ($z = x + iy$). So the first step is to convert these complex numbers from polar to rectangular form. We have

\[
\begin{align*}
\text{Re}(a) &= 2.0 \cos(\pi/2) = 0.000 \\
\text{Im}(a) &= 2.0 \sin(\pi/2) = 2.000 \\
\text{Re}(b) &= 3.0 \cos(\pi/3) = 1.500 \\
\text{Im}(b) &= 3.0 \sin(\pi/3) = 2.598
\end{align*}
\]

Remember to use radians to calculate the sine and cosine. To summarize we now have

\[
\begin{align*}
a &= 2.0 \exp(i \pi/2) = 0.000 + 2.000i \\
b &= 3.0 \exp(i \pi/3) = 1.500 + 2.598i
\end{align*}
\]

Now we can add them.

\[
a + b = (0.000 + 1.500) + (2.000 + 2.598)i = 1.500 + 4.598i
\]

The magnitude of this complex number is

\[
\text{mag}(c) = \sqrt{(\text{Re}(c))^2 + (\text{Im}(c))^2} = \sqrt{(1.500)^2 + (4.598)^2} = 4.836
\]

By the way, the angle associated with this complex number is

\[
\text{ang}(c) = \tan^{-1}\left(\frac{\text{Im}(c)}{\text{Re}(c)}\right) = \tan^{-1}\left(\frac{4.598}{1.500}\right) = 1.255
\]

So $c$ in polar form is

\[
c = (4.836) \exp(1.255i)
\]
1. An airplane wing is designed so that the speed of the air across the top of the wing is 251 m/s when the speed of the air below the wing is 225 m/s. The density of the air is 1.29 kg/m$^3$ What is the lifting force on a wing of area 24.0 m$^2$?

**Solution**

Bernoulli’s equation:

$$\Delta P = \rho gh + \frac{1}{2}\rho(v_h^2 - v_l^2)$$

In this case the height differential $h$ is negligible, so we have:

$$\Delta P = \frac{1}{2}\rho(v_h^2 - v_l^2)$$

And substituting our data, we have:

$$\Delta P = \frac{1}{2}(1.29)((251)^2 - (225)^2) = 7982.5$$

Since pressure is force over area, the total lifting force is this pressure differential times the surface area:

$$F = (7982.5)(24.0) = 191580$$

2. A waterbed for sale has dimensions of 1.83 meters by 2.13 meters by 0.229 meters. The floor of the bedroom will tolerate an additional weight of no more than 6660 newtons. Find the weight of the water in the bed and determine whether the bed should be purchased.

**Solution**

The volume of the water bed is

$$V = (1.83)(2.13)(0.229) = 0.89262$$

Since the density of water is 1000 kg/m$^3$, this implies that

$$m = \rho V = (1000)(0.89262) = 892.62$$

Which weighs

$$W = mg = (892.62)(9.80) = 8747.7$$

The waterbed will fall through the bedroom floor! Might want to keep looking for a new bed.

3. A person who weighs 625 newtons is riding a 98-newton mountain bike. Suppose the entire weight of the rider and bike is supported equally by the two tires. If the gauge pressure in each tire is 760 kPa, what is the area of contact between each tire and the ground?

**Solution**

By definition, pressure is defined as $P = F/A$. In this problem we know both the force involved (half of 625 plus 98) and the pressure. The only trick is to remember that “gauge” pressure is calibrated to be zero under atmospheric pressure, so really one must add an additional 101.3 kPa to get the absolute pressure here. In the end we have:

$$(861300) = \frac{361.5}{A} \implies A = 4.1971 \times 10^{-3}$$

4. A submersible pump is put under the water at the bottom of a well and is used to push water up through a pipe. What minimum output gauge pressure 700 kilopascals
must the pump generate to make the water reach the nozzle at ground level, 71 meters above the pump?

Solution

The pressure differential in any water column is given by \( \Delta P = \rho gh \). In this case we have

\[
\Delta P = (1000)(9.80)(71) = 695800
\]

Now, to get the absolute pressure at the bottom we need to take into account the atmospheric pressure at the top. But we are asked for the gauge pressure instead, so we need to back this correction back out. So the final calculation still yields 695,800 Pa.

5. A water tower is a familiar sight in many towns. The purpose of such a tower is to provide storage capacity and to provide sufficient pressure in the pipes that deliver the water to customers. Figure 2 shows a spherical reservoir that contains \( 5.25 \times 10^5 \) kilograms of water when full. The reservoir is vented to the atmosphere at the top. For a full reservoir, find the gauge pressure that the water has at the faucet in (a) house A and (b) house B. Ignore the diameter of the delivery pipes.

Figure 2: Problem 11.27

Solution

(a) We need to know the height of the spherical reservoir to use the basic hydrostatic equation \( \Delta P = \rho gh \). Fortunately we are told that the reservoir is a sphere.\footnote{Remember that the pressure differential \textit{only} depends on the height and not the shape of the container. Whether a pipe or a sphere, only the height matters.} If we knew its volume, we could determine its radius. But the volume is connected to the weight of water (which we know) through the density of water. By definition, density is \( \rho = M/V \), so

\[
(1000) = \frac{5.25 \times 10^5}{V} \implies V = 525
\]

The radius is can be derived from \( V = \frac{4}{3} \pi r^3 \):

\[
(525) = \frac{4}{3} \pi (r)^3 \implies r = 5.0045
\]

So, the height of the reservoir is

\[
2r = 10.009
\]

So, the total height from the top (exposed to air) and the bottom is

\[
2r + 15.0 = 25.009
\]

This height introduces a pressure differential of

\[
\Delta P = \rho gh = (1000)(9.80)(25.009) = 245088
\]

(b) 245 kPa

(b) 174 kPa
This is the gauge pressure because it is the differential relative to atmospheric pressure (because the top of the reservoir is exposed to air).

(b) The extra height of 7.30 meters reduces the water pressure based on the same formula, ΔP = ρgh.

\[ \Delta P = (1000)(9.80)(7.30) = 71540 \]

So the gauge pressure at the elevated house is

\[ P = 245088 - 71540 = 173548 \]

6. What is the radius of a hydrogen-filled balloon that would carry a load of 5750 newtons (in addition to the weight of the hydrogen) when the density of air is 1.29 kg/m³?

Solution

We are looking for a buoyant force that will lift both 5750 newtons of weight and the weight of the hydrogen in the balloon. According to Archimedes’ principle, this buoyant force is equal to the weight of the displaced fluid (in this case air).

In symbols,

\[ W_{\text{air}} = 5750 + W_{\text{hydrogen}} \]

Now, both weights are related to volume through mass and density. In other words,

\[ W = \rho V g \]

Since the density of hydrogen gas is 0.0899 kg/m³, we have:

\[ (1.29)(V)(9.80) = (5750) + (0.0899)(V)(9.80) \implies V = 488.90 \]

But, if this volume is in the shape of a sphere, its radius can be gotten through

\[ V = \frac{4}{3} \pi r^3 \]. Thus,

\[ (488.90) = \left(\frac{4}{3}\pi\right)(r)^3 \implies r = 4.8870 \]

7. A lighter-than-air balloon and its load of passengers and ballast are floating stationary above the earth. Ballast is weight (of negligible volume) that can be dropped overboard to make the balloon rise. The radius of this balloon is 6.25 meters. Assuming a constant value of 1.29 kg/m³ for the density of air, determine how much weight must be dropped overboard to make the balloon rise 105 meters in 15.0 seconds.

Solution

Before the ballast is dropped, the balloon is in equilibrium—its weight is supported by an equal buoyant force. After the ballast is dropped, the buoyant force exceeds the left-over weight producing a net force and acceleration upward. We can determine that acceleration from the data given:

\[ v_0 = 0 \]
\[ x = 105 \]
\[ t = 15.0 \]
\[ a = ? \]

The equation we need is \( x = v_0t + \frac{1}{2}at^2 \). Thus,

\[ (105) = (0)(15.0) + \frac{1}{2}(a)(15.0)^2 \implies a = 0.9333 \]

If we knew the mass of the balloon, we could figure out the net force on the balloon via Newton’s second law. Let’s call the mass of the balloon \( M \) and the mass of the ballast \( m \). Thus,

\[ F_{\text{net}} = (M)(0.9333) \]
And we know that the net force is the bouyant force minus the weight of the balloon. We also know that the bouyant force is equal to the weight of the displaced fluid (Archimedes’ principle). In this case we can calculate the displaced volume (using $V = \frac{4}{3}\pi r^3$), mass (using $\rho = M/V$), and weight (using $W = mg$). So the volume is

$$V = \frac{4}{3}\pi (6.25)^3 = 1022.7$$

and the mass is

$$(1.29) = \frac{(M)}{(1022.7)} \implies M = 1319.3$$

and the weight is

$$W = mg = (1319.3)(9.80) = 12929$$

This is also the magnitude of the bouyant force. This means that the net force on the balloon (without the ballast) is

$$F_{\text{net}} = (12929) - (M)(9.80)$$

Putting this together with Newton’s second law, we have:

$$(M)(0.9333) = (12929) - (M)(9.80) \implies M = 1204.6$$

This is good, but not enough. What we really want the mass of with the ballast. What else do we know? We know that prior to the ballast being dropped, the forces were in equilibrium. Dropping the ballast doesn’t affect the bouyant force. The total mass with the ballast is $(1204.6 + m)$, so we have:

$$(12929) = (1204.6 + m)(9.80) \implies m = 114.72$$

Which means that the weight of the ballast is

$$W = mg = (114.72)(9.80) = 1124.3$$

8. A fuel pump sends gasoline from a car’s fuel tank to the engine at a rate of $0.0588$ kg/s. The density of the gasoline is $735$ km/m$^3$ and the radius of the fuel line is $3.18$ mm. What is the speed at which the gasoline moves through the fuel line?

Solution

We need to determine the volume flow rate, $Q$. Once we know this number we can use $Q = Av$ to determine the speed of the gasoline. We are told the mass flow rate. But mass and volume are connected through density. Since $m = \rho V$, the rates are related in the same way. Therefore,

$$(0.0588) = (735)(Q) \implies Q = 8.0000 \times 10^{-5}$$

Now we need to calculate the cross-sectional area of the fuel line:

$$A = \pi r^2 = (\pi)(0.00318)^2 = 3.1769 \times 10^{-5}$$

Finally, we use $Q = Av$:

$$(8.0000 \times 10^{-5}) = (3.1769 \times 10^{-5})(v) \implies v = 2.5182$$

9. Water is circulating through a closed system of pipes in a two-floor apartment. On the first floor, the water has a gauge pressure of $340$ kPa and a speed of $2.1$ m/s. However, on the second floor, which is $4.0$ meters higher, the speed of the water is $3.7$ m/s. The speeds are different because the pipe diameters are different. What is the gauge pressure of the water on the second floor?

Solution

Water is circulating through a closed system of pipes in a two-floor apartment. On the first floor, the water has a gauge pressure of $340$ kPa and a speed of $2.1$ m/s. However, on the second floor, which is $4.0$ meters higher, the speed of the water is $3.7$ m/s. The speeds are different because the pipe diameters are different. What is the gauge pressure of the water on the second floor?

Solution

Water is circulating through a closed system of pipes in a two-floor apartment. On the first floor, the water has a gauge pressure of $340$ kPa and a speed of $2.1$ m/s. However, on the second floor, which is $4.0$ meters higher, the speed of the water is $3.7$ m/s. The speeds are different because the pipe diameters are different. What is the gauge pressure of the water on the second floor?

Solution
Bernoulli’s equation:
\[ \Delta P = \rho g h + \frac{1}{2} \rho (v_f^2 - v_i^2) \]

Plugging in our data:
\[ \Delta P = (1000)(9.80)(4.0) - \frac{1}{2}(1000)((3.7)^2 - (2.1)^2) = 43840 \]

This \( \Delta P \) represents the amount of increase with depth. In symbols:
\[ \Delta P = P_l - P_h \]

Thus,
\[ 43840 = 340000 - P_h \implies P_h = 296160 \]

10. A blood transfusion is being set up in an emergency room for an accident victim. Blood has a density of 1060 kg/m\(^3\) and a viscosity of \( \eta \) is \( 4.0 \times 10^{-3} \) Pa-s. The needle being used has a length of 3.0 cm and an inner radius of 0.25 mm. The doctor wishes to use a volume flow rate through the needle of \( 4.5 \times 10^{-8} \) m\(^3\)/s. What is the distance \( h \) above the victim’s arm where the level of the blood in the transfusion bottle should be located? As an approximation, assume that the level of the blood in the transfusion bottle and the point where the needle enters the vein in the arm have the same pressure of one atmosphere. (In reality, the pressure in the vein is slightly above atmospheric pressure.)

Solution

This is an application of Poiseuille’s law:
\[ Q = \frac{\pi r^4 \Delta P}{8 \eta L} \]

Plugging in what we have
\[ (4.5 \times 10^{-8}) = \frac{(\pi)(0.25 \times 10^{-3})^4(\Delta P)}{(8)(4.0 \times 10^{-3})(0.030)} \implies \Delta P = 3520.3 \]

This is the pressure differential that will create the required volume flow rate out of the needle. It remains to calculate the height that will create this pressure differential. Since the volume flow rate is so small, this can be done using the hydrostatic equation, \( \Delta P = \rho gh \). Thus,
\[ 3520.3 = (1060)(9.80)(h) \implies h = 0.33888 \]
1. One end of an iron poker is placed in a fire where the temperature is 502 °C, and the other end is kept at a temperature of 26 °C. The poker is 1.2 meters long and has a radius of 5.0 millimeters. Ignoring the heat lost along the length of the poker, find the amount of heat conducted from one end of the poker to the other in 5.0 seconds.

**Solution**

The equation for the conduction of heat is

\[ P = \frac{Q}{t} = \frac{k A}{L} \Delta T \]

The thermal conductivity, \( k \), of iron is 79. The temperature differential in the poker is 502 - 26 = 476. In addition, the cross-sectional area of the poker is

\[ A = \pi r^2 = (\pi)(5.0 \times 10^{-3})^2 = 7.8240 \times 10^{-5} \]

Plugging all this in, we have:

\[ \frac{Q}{5.0} = \frac{(79)(7.8240 \times 10^{-5})}{1.2}(476) \implies Q = 12.259 \]

2. A steel aircraft carrier is 370 meters long when moving through the icy North Atlantic at a temperature of 2.0 °C. By how much does the carrier lengthen when it is traveling in the warm Mediterranean Sea at a temperature of 21 °C?

**Solution**

The coefficient of thermal expansion of steel is given in Table 12.1 of the book as \( 12 \times 10^{-6} \)°C. The temperature differential in this situation is 19 °C. Using the equation for thermal expansion, \( \Delta L = L_0 \alpha \Delta T \), we have:

\[ \Delta L = (12 \times 10^{-6})(370)(19) = 0.084360 \]

3. A 0.35-kilogram coffee mug is made from a material that has a specific heat capacity of 920 J/kg·°C and contains 0.25 kilograms of water. The cup and water are at 15 °C. To make a cup of coffee, a small electric heater is immersed in the water and brings it to a boil in three minutes. Assume that the cup and water always have the same temperature and determine the minimum power rating of this heater.

**Solution**

In order to determine the power used, we need to know the total energy used and divide by the time interval (three minutes, or 180 seconds). The energy is the heat required to warm both the water and the cup 85 °C to the boiling point of water. The relevant equation to use is \( \Delta Q = cm \Delta T \). The amount of heat to warm the water is

\[ \Delta Q = (4186)(0.25)(85) = 88953 \]

And the amount of heat required to warm the cup is

\[ \Delta Q = (920)(0.35)(85) = 27370 \]

So, the total heat used is

\[ \Delta Q = (88953) + (27370) = 116320 \]
So, the minimum power rating is

\[ P = \frac{\Delta E}{\Delta t} = \frac{(116320)}{(180)} = 646.24 \]

Since there will be some heat lost to the surrounding air and such, this represents the theoretical minimum power required.

4. To help prevent frost damage, fruit growers sometimes protect their crop by spraying it with water when overnight temperatures are expected to go below the freezing markelvin. When the water turns to ice during the night, heat is released into the plants, thereby giving them a measure of protection against the falling temperature. Suppose a grower sprays 7.2 kilograms of water at 0 °C onto a fruit tree. (a) How much heat is released by the water when it freezes? (b) How much would the temperature of a 180-kilogram tree rise if it absorbed the heat released in part (a)? Assume that the specific heat capacity of the tree is 2500 J/kg-°C and that no phase change occurs within the tree itself.

Solution

(a) The heat of fusion for water is 33.5 × 10^4 J/kg. Since there are 7.2 kilograms of water, the total heat released is

\[ \Delta Q = mL_f = (7.2)(33.5 \times 10^4) \implies \Delta Q = 2.4120 \times 10^6 \]

(b) When this heat flows into the tree, its temperature increases according to \( \Delta Q = cm\Delta T \). Thus,

\[ (2.4120 \times 10^6) = (2500)(180)(\Delta T) \implies \Delta T = 5.3600 \]

5. A woman finds the front windshield of her car covered with ice at -12.0 °C. The ice has a thickness of 0.450 mm, and the windshield has an area of 1.25 m^2. The density of ice is 917 kg/m^3. How much heat is required to melt the ice?

Solution

Since the ice is at a temperature below its melting point, we will have to heat up the ice before it will melt. In both of these calculations we will need to know the mass of the ice involved. We are told the density and can calculate the volume, so we should be able to determine the mass. The volume is

\[ V = (1.25)(0.450 \times 10^{-3}) = 5.6250 \times 10^{-4} \]

and using the density formula \( \rho = M/V \), we have:

\[ (917) = \frac{M}{(5.6250 \times 10^{-4})} \implies M = 0.51581 \]

We want to increase the temperature 12 °C to get to the melting point. The relevant equation is \( \Delta Q = cm\Delta T \), so

\[ \Delta Q = (2000)(0.51581)(12) = 12380 \]

And the heat required to melt this ice is given by the equation \( \Delta Q = mL_f \):

\[ \Delta Q = (0.51581)(33.5 \times 10^4) = 172800 \]

The total heat is the sum of these two numbers:

\[ \Delta Q = (12380) + (172800) = 185180 \]

6. Ice at -10.0 °C and steam at 130 °C are brought together at atmospheric pressure in a perfectly insulated container. After thermal equilibrium is reached,
the liquid phase at 50.0 °C is present. Ignoring the container and the equilibrium vapor pressure of the liquid at 50.0 °C, find the ratio of the mass of steam to the mass of ice. The specific heat capacity of steam is 2020 J/(kg·°C).

Solution

We don’t know the masses involved here, but we will need to know them, so let’s give them some labels. Let’s call the mass of ice \( m_1 \) and the mass of steam \( m_2 \). As far as the ice is concerned, we know that heat is required to bring the temperature of the ice up to its melting point, then melt it, then raise the temperature of the liquid ice (i.e., water) up to 50 °C. So there are three steps to consider. Let’s take them one at a time. First, raise the temperature of the ice:

\[
\Delta Q = cm\Delta T = (2000)(m_1)(10) = (20000)(m_1)
\]

Then melt the ice:

\[
\Delta Q = mL_f = (m_1)(335000)
\]

Then raise the temperature of the liquid ice:

\[
\Delta Q = cm\Delta T = (4186)(m_1)(50) = (209300)(m_1)
\]

Adding these together yields the total heat required:

\[
\Delta Q = (20000)(m_1) + (335000)(m_1) + (209300)(m_1) = (564300)(m_1)
\]

This heat will come from the steam. The temperature of the steam must be brought down to its liquefaction point, then the steam must be liquified, then the liquid steam (i.e., water) must cool down to 50 °C—another three step process. Thus,

\[
\Delta Q = cm\Delta T = (2020)(m_2)(30) = (60600)(m_1)
\]

Then melt the ice:

\[
\Delta Q = mL_f = (m_2)(2260000)
\]

Then raise the temperature of the liquid ice:

\[
\Delta Q = cm\Delta T = (4186)(m_2)(50) = (209300)(m_2)
\]

Adding these together yields the total heat transferred:

\[
\Delta Q = (60600)(m_2) + (2260000)(m_2) + (209300)(m_2) = (2529900)(m_2)
\]

Okay. These two totals must be equal to one another. Thus,

\[
(564300)(m_1) = (2529900)(m_2)
\]

But we want \( m_2/m_1 \). Thus,

\[
\frac{m_2}{m_1} = \frac{564300}{2529900} = 0.22305
\]

7. A wall in a house contains a single window. The window consists of a single pane of glass whose area is 0.16 m² and whose thickness is 2.0 mm. Treat the wall as a slab of the insulating material Styrofoam whose area and thickness are 18 m² and 0.10 meters, respectively. Heat is lost via conduction through the window and the wall. The temperature difference between the inside and outside is the same for the window and the wall. Of the total heat lost by the wall and the window, what is the percentage lost by the window?

Solution

The formula for the rate of the conduction of heat is

\[
P = \frac{Q}{t} = \frac{kA}{L} \Delta T
\]
For the glass, we have an overall thermal conductivity of
\[
\frac{kA}{L} = \frac{(0.80)(0.16)}{(0.0020)} = 64.000
\]
And for the wall, we have
\[
\frac{kA}{L} = \frac{(0.010)(18)}{(0.10)} = 1.8000
\]
The total thermal conductivity is the sum of these two, 65.800 W/°C. Therefore, the percent of the total heat lost by the window is
\[
\frac{64.000}{65.800} = 0.97264
\]

8. In an aluminum pot, 0.15 kilograms of water at 100 °C boils away in four minutes. The bottom of the pot is 3.1 mm thick and has a surface area of 0.015 m². To prevent the water from boiling too rapidly, a stainless steel plate has been placed between the pot and the heating element. The plate is 1.4 mm thick, and its area matches that of the pot. Assuming that heat is conducted into the water only through the bottom of the pot, find the temperature at (a) the aluminum-steel interface and (b) the steel surface in contact with the heating element.

Solution
(a) The rate at which the heat is flowing into the water is
\[
P = \frac{\Delta Q}{\Delta t}
\]
We know the time frame is four minutes (or 240 seconds). The amount of heat that flows is given by
\[
\Delta Q = mL_v = (0.15)(22.6 \times 10^5) = 3.3900 \times 10^5
\]
Thus,
\[
P = \frac{(3.3900 \times 10^5)}{(240)} = 1412.5
\]
This power flow is driven by a temperature differential between the water (100 °C) and the other side of the aluminum. This conduction is governed by the equation
\[
P = \frac{Q}{t} = \frac{kA}{L} \Delta T
\]
For the aluminum plate, the total conductance is
\[
\frac{kA}{L} = \frac{(240)(0.015)}{(3.1 \times 10^{-3})} = 1161.3
\]
Plugging this into the conduction equation gives
\[
(1412.5) = (1161.3)(\Delta T) \implies \Delta T = 1.2163
\]
Since the water is at 100 °C, the other side of the aluminum is
\[
T = 100 + 1.12163 = 101.12
\]
(b) The power that flows into the water through the aluminum plate also flows through the steel plate. The total conductance of the steel plate is
\[
\frac{kA}{L} = \frac{(14)(0.015)}{(1.4 \times 10^{-3})} = 150.00
\]
So, the conductance equation for the steel plate is
\[(1412.5) = (150.00)(\Delta T) \implies \Delta T = 9.4167\]
This is the temperature differential from the aluminum plate: 101.22 °C. So, the temperature on the other side of the steel plate is
\[T = 101.12 + 9.4167 = 110.54\]

9. Liquid helium is stored at its boiling-point temperature of 4.2 kelvin in a spherical container (radius = 0.30 meters). The container is a perfect blackbody radiator. The container is surrounded by a spherical shield whose temperature is 77 kelvin. A vacuum exists in the space between the container and the shield. The latent heat of vaporization for helium is 21000 J/kg. What mass of liquid helium boils away through a venting valve in one hour?

**Solution**

The key formula in this problem is
\[P = \frac{Q}{t} = (\varepsilon \sigma A)(T^4)\]
We will need to know the surface area of the container. It is
\[A = 4\pi r^2 = (4\pi)(0.30)^2 = 1.1310\]
The trick to remember about the radiation formula is that it only represents the flow of heat in one direction. Heat is flowing into the helium from the surroundings due to this external temperature but it is also losing heat due to radiation based on its internal temperature.

The rate of heat that flows into the helium is
\[P = (1)(5.67 \times 10^{-8})(1.1310)(77)^4 = 2.2542\]
And the amount of heat lost from the helium is
\[P = (1)(5.67 \times 10^{-8})(1.1310)(4.2)^4 = 1.9954 \times 10^{-5}\]
Now, this is negligible compared to the heat flowing into the helium. In the time frame mentioned (one hour = 3600 seconds), the total heat that flows is
\[(2.2542) = \frac{\Delta Q}{3600} \implies Q = 8115.0\]
The vaporization is governed by the equation \(\Delta Q = mL_v\), so
\[(8115.0) = (m)(21000) \implies m = 0.38643\]

10. The amount of radiant power produced by the sun is approximately \(3.9 \times 10^{26}\) watts. Assuming the sun to be a perfect blackbody sphere with a radius of \(6.96 \times 10^8\) meters, find its surface temperature (in kelvin).

**Solution**

The radiant power is given by
\[P = \frac{Q}{t} = (\varepsilon \sigma A)(T^4)\]
Since we are treating this as a blackbody, \(\varepsilon\) is one. The last thing we need to determine is the surface area of the sun. The formula is \(A = 4\pi r^2\), so:
\[A = (4\pi)(6.96 \times 10^8)^2 = 6.0873 \times 10^{18}\]
Plugging all this in we have:
\[(3.9 \times 10^{26}) = (1)(5.67 \times 10^{-8})(6.0873 \times 10^{18})(T^4)\]
\[\implies T = 5797.8\]
Physics 202 Homework 4  
Apr 24, 2013

1. In a diesel engine, the piston compresses air at 305 K to a volume that is one-sixteenth of the original volume and a pressure that is 48.5 times the original pressure. What is the temperature of the air after the compression?

Solution

Here we need to use the ideal gas law

\[ PV = nRT \]

If we apply this law to the initial and final states, then divide the two equations we get:

\[ \frac{P_2V_2}{P_1V_1} = \frac{T_2}{T_1} \]

The \( nR \) cancels because \( R \) is a constant, and the amount of gas (\( n \)) does not change. In the statement of the problem we are told that \( \frac{V_2}{V_1} = \frac{1}{16} \) and \( \frac{P_2}{P_1} = 48.5 \). Thus,

\[ \left( \frac{1}{16} \right) (48.5) = \frac{T_2}{305} \implies T_2 = 924.53 \]

2. Suppose that a tank contains 680 m\(^3\) of neon at an absolute pressure of 101 kPa. The temperature is changed from 293.2 to 294.3 kelvin. What is the increase in the internal energy of the neon?

Solution

The formula for internal energy of a monoatomic gas (like neon) is

\[ \Delta U = \frac{3}{2} nR \Delta T \]

The only hard part here is to determine \( n \). But we can get that from the ideal gas law. Initially, we have

\[ PV = nRT \implies (1.01 \times 10^5)(680) = (n)(8.31)(293.2) \]

\[ \implies n = 28188 \]

Since the temperature differential is 294.3 - 293.2 = 1.1, we have

\[ \Delta U = \frac{3}{2}(28188)(8.31)(1.1) = 3.865 \times 10^5 \]

3. A certain element has a mass per mole of 196.967 g/mol. What is the mass of a single atom in (a) atomic mass units and (b) kilograms? (c) How many moles of atoms are in a 285-gram sample?

Solution

(a) The conversion from atomic mass unit (per molecule) to grams per mole is one-to-one. This is really the main advantage to using atomic mass units when dealing with nature at the atomic level. So, no conversion is necessary.

(b) There are 0.196967 kilograms per mole. Since there are \( 6.022 \times 10^{23} \) molecules in a mole, the mass per molecule is

\[ \frac{0.196967 \text{ kg}}{\text{mole}} \times \frac{1 \text{ mole}}{6.022 \times 10^{23} \text{ molecules}} = 3.2708 \times 10^{-25} \]

(c) This is easier if we go back to the original metric of grams per mole. Thus,

\[ 285 \text{ g} \times \frac{1 \text{ mol}}{196.967 \text{ g}} = 1.4469 \text{ mol} \]
4. An ideal gas at 15.5 °C and a pressure of 172 kPa occupies a volume of 2.81 m³. (a) How many moles of gas are present? (b) If the volume is raised to 4.16 m³ and the temperature raised to 28.2 °C, what will be the pressure of the gas?

Solution
(a) The ideal gas law states $PV = nRT$. Thus,

$$ (172000)(2.81) = (n)(8.31)(15.5 + 273.15) \implies n = 201.49 $$

Notice the conversion to kelvin. This is one of the formulas in which it is necessary to always use kelvin.

(b) Using the ideal gas law again,

$$ (P)(4.16) = (201.49)(8.31)(28.2 + 273.15) \implies P = 121292 $$

5. A bubble, located 0.200 m beneath the surface in a glass of beer, rises to the top. The air pressure at the top is 101 kPa. Assume that the density of beer is the same as that of fresh water. If the temperature and number of moles of CO₂ remain constant as the bubble rises, find the ratio of its volume at the top to that at the bottom.

Solution
Let’s call the volume at the bottom of the beer $V_1$. Since the pressure at the bottom is under a fluid, there is an incremental pressure due to the bubble’s depth. This additional pressure is given by the hydrostatic equation $\Delta P = \rho gh$. In this case,

$$ \Delta P = (1000)(9.80)(0.200) = 1960 $$

Since the top of the beer is near atmospheric pressure, the bottom is at

$$ P_1 = 101000 + 1960 = 102960 $$

The ideal gas law, $PV = nRT$, tells us

$$ (102960)(V_1) = nRT $$

Since we are assuming $n$ and $T$ stay the same, the right-hand-side of this equality is the same at the top, too:

$$ (101000)(V_2) = nRT $$

Where $V_2$ is the volume at the top of the beer. We can set the two left-hand-sides together to yield:

$$ (102960)(V_1) = (101000)(V_2) $$

We are asked for the ratio $V_2/V_1$. We can rearrange the equation above to get:

$$ \frac{V_2}{V_1} = \frac{102960}{101000} = 1.0194 $$

There is a 1.9% increase in the size of the bubble due to the pressure difference.

6. In 10.0 seconds, 200 bullets strike and embed themselves in a wall. The bullets strike the wall perpendicularly. Each bullet has a mass of 5.0 grams and a speed of 1200 m/s. (a) What is the average change in momentum per second for the bullets? (b) Determine the average force exerted on the wall (c) Assuming the bullets are spread out over an area of $3.0 \times 10^{-4}$ m², obtain the average pressure they exert on this region of the wall.

Solution
(a) Each bullet has an initial momentum of

$$ p = mv = (0.0050)(1200) = 6.0000 $$

(b) 120 kg-m/s per second
(c) 120 newtons (c) 400 kPa

1.02

18
The bullets embed themselves in the wall, so the final momentum is zero. For 200 bullets, this is a change in momentum of
\[ \Delta p = (200)(6.0000) = 1200.0 \]
Divide by the timeframe of 10.0 seconds to get the change in momentum per second:
\[ \frac{\Delta p}{\Delta t} = \frac{1200.0}{10.0} = 120.00 \]
(b) The impulse momentum theorem states that \( F \Delta t = \Delta p \). In this case we have:
\[ (F)(10.0) = (1200.0) \implies F = 120.00 \]
(c) By definition, pressure is force divided by area, so
\[ P = \frac{F}{A} = \frac{120.00}{3.0 \times 10^{-4}} = 400000 \]

7. Carbon tetrachloride (CCl\(_4\)) is diffusing through benzene (C\(_6\)H), as Figure 3 illustrates. The concentration of CCl\(_4\) at the left end of the tube is maintained at 0.0100 kg/m\(^3\), and the diffusion constant is 20.0 \( \times \) 10\(^{-10} \) m\(^2\)/s. The CCl\(_4\) enters the tube at a mass rate of 5.00 \( \times \) 10\(^{-13} \) kg/s. Using these data and those shown in the drawing, find (a) the mass of CCl\(_4\) per second that passes point A and (b) the concentration of CCl\(_4\) at point A.

![Figure 3: Problem 14.50](image)

**Solution**

(a) Due to the equation of continuity, the mass flow rate is the same at all points in the fluid. So, the mass flow rate is the same everywhere: 5.00 \( \times \) 10\(^{-13} \).

(b) Fick’s law of diffusion states
\[ \frac{m}{t} = DA \frac{\Delta C}{L} \]
In this case, the quantity \( DA/L \) is
\[ \frac{(20.0 \times 10^{-10})(3.00 \times 10^{-4})}{5.00 \times 10^{-3}} = 1.2000 \times 10^{-10} \]
Plugging this into Fick’s law:
\[ (5.00 \times 10^{-13}) = (1.2000 \times 10^{-10})(\Delta C) \implies \Delta C = 4.1667 \times 10^{-3} \]
Since the concentration at the left end of the tube is 1.00 \( \times \) 10\(^{-2} \), the concentration at point A must be:
\[ (1.0 \times 10^{-2}) - (4.1667 \times 10^{-3}) = 5.8333 \times 10^{-3} \]

8. At the start of a trip, a driver adjusts the absolute pressure in her tires to be 304 kelvin
281 kPa when the outdoor temperature is 284 kelvin. At the end of the trip she measures the pressure to be 301 kPa. Ignoring the expansion of the tires, find the air temperature inside the tires at the end of the trip.

Solution

In both cases we use the ideal gas law \( PV = nRT \). We can apply this law to the initial and final states, then divide the two equations we get:

\[
\frac{P_2}{P_1} = \frac{T_2}{T_1}
\]

The \( V \) cancels because we are ignoring the expansion of the tires, so the volume is essentially held constant. The \( n \) cancels since no additional gas is added or lost and the \( R \) is a constant. Plugging in our data yields

\[
\frac{301000}{281000} = \frac{T_2}{284} \implies T_2 = 304.21\text{ K}
\]

9. Estimate the spacing between the centers of neighboring atoms in a piece of solid aluminum, based on a knowledge of the density (2700 kg/m\(^3\)) and atomic mass (26.9815 u) of aluminum. (Hint: Assume that the volume of the solid is filled with many small cubes, with one atom at the center of each.)

Solution

Given the density and the atomic mass, we can calculate the atomic volume. But we need the atomic mass converted to kilograms first. Since the atomic mass is also the number grams per mole, we have 0.0269815 kilograms per mole of aluminum. But we want per molecule (or atom, in this case). Thus,

\[
0.0269815 \text{ kg/mol} \times \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ molecules}} = 4.4805 \times 10^{-26} \text{ kg/molecule}
\]

Using the density formula, \( \rho = M/V \), we have

\[
(2700) = \frac{4.4805 \times 10^{-26}}{V} \implies V = 1.6594 \times 10^{-29} \text{ m}^3
\]

Assuming the atoms stack like cubes, the side of these cubes must be

\[
l = \sqrt[3]{1.6594 \times 10^{-29}} = 2.5507 \times 10^{-10} \text{ m}
\]

10. The mass of a hot-air balloon and its occupants is 320 kg (excluding the hot air inside the balloon). The air outside the balloon has a pressure of 101 kPa and a density of 1.29 kg/m\(^3\). To lift off, the air inside the balloon is heated. The volume of the heated balloon is 650 m\(^3\). The pressure of the heated air remains the same as that of the outside air. To what temperature (in kelvins) must the air be heated so that the balloon just lifts off? The molecular mass of air is 29 u.

Solution

Remember, that the buoyant force is the weight of the fluid that has been displaced. In this case, we know that volume and the density of the fluid, so we can calculate the mass using \( m = \rho V \). Given the displaced mass, the displaced weight is simply \( W = mg \). Thus,

\[
W = (1.29)(650)(9.80) = 8217.3
\]

The weight of the balloon and occupants is

\[
W = (320)(9.80) = 3136.0
\]

Which means that the weight of the hot air in the balloon must be less than

\[
8217.3 - 3136.0 = 5081.3
\]
This corresponds to a mass of

\[ W = mg \implies (5081.3) = (m)(9.80) \implies m = 518.50 \]

We will use this to determine the number of moles of air in the balloon. Since the molecular mass is 29, we know that

\[ m = (0.029)(n) \]

This is because a molecular mass of 29 implies that there are 0.029 kilograms in each mole. Thus,

\[ (518.50) = (0.029)(n) \implies n = 17879 \]

This is where we pull out the ideal gas law, \( PV = nRT \). Since the balloon is open to air, the pressure inside is also 101 kPa. We know the volume and number of moles. Thus,

\[ (101000)(650) = (17879)(8.31)(T) \implies T = 441.86 \]

This is about 168 °C, or 336 °F.
1. A nuclear-fueled electric power plant utilizes a so-called “boiling water reactor.” In this type of reactor, nuclear energy causes water under pressure to boil at 285 °C (the temperature of the hot reservoir). After the steam does the work of turning the turbine of an electric generator, the steam is converted back into water in a condenser at 40 °C (the temperature of the cold reservoir). To keep the condenser at 40 °C, the rejected heat must be carried away by some means—for example, by water from a river. The plant operates at three-fourths of its Carnot efficiency, and the electrical output power of the plant is 1.2 × 10⁹ watts. A river with a water flow rate of 1.0 × 10⁵ kg/s is available to remove the rejected heat from the plant. Find the number of Celsius degrees by which the temperature of the river rises.

Solution

The Carnot efficiency of the plant is
\[ e_{\text{rev}} = 1 - \frac{T_c}{T_h} = 1 - \frac{273.15 + 40}{273.15 + 285} = 0.43895 \]

But the plant operates at 75% of the efficiency, so its true efficiency is
\[ e = (0.75)(e_{\text{rev}}) = 0.3292 \]

By definition, the efficiency is
\[ e = \frac{\Delta W}{\Delta Q_h} \]

Since we know the output power we know that the work done each second is 1.2 × 10⁹ joules. This allows us to calculate the input heat coming from the nuclear reactions:
\[ 0.3292 = \frac{1.2 \times 10^9}{\Delta Q_h} \implies \Delta Q_h = 3.6452 \times 10^9 \]

But it is the waste heat that is rejected into the river. That is:
\[ \Delta Q_e = \Delta Q_h - \Delta W = 3.6452 \times 10^9 - 1.2 \times 10^9 = 2.4452 \times 10^9 \]

This is the amount of heat that goes into the flowing river. We know that 1.0 × 10⁵ kg of water flow past the plant each second. We need to determine how much the heat causes its temperature to rise. We use \[ \Delta Q = cm\Delta T \]. Thus,
\[ 2.4452 \times 10^9 = (4186)(1.0 \times 10^5)(\Delta T) \implies \Delta T = 5.84 \]

2. Three moles of an ideal gas are compressed from 0.055 to 0.025 m³. During the compression, 6100 joules of work is done on the gas, and heat is removed to keep the temperature of the gas constant at all times. Find (a) \( \Delta U \), (b) \( Q \), and (c) the temperature of the gas.

Solution

(a) Notice that this is an isothermal compression. The change in internal energy of an ideal gas is proportional to its change in temperature (e.g., \( \Delta U = \frac{3}{2}nR\Delta T \) for a monoatomic gas). Since there is no change in temperature, there is no change in internal energy.

(b) The first law states that both heat and work change the internal energy of a system. Since we know there is in fact no change in internal energy, the energy done performing work on the gas must go out as the heat removed, so
\[ \Delta Q = -\Delta W = -6100 \]
(c) The work done in an isothermal compression is given by
\[ \Delta W = nRT \ln \left( \frac{V_f}{V_i} \right) \]
Since we know the work done, we can extract the temperature of the gas from this equation.
\[-6100 = (3)(8.31)(T) \ln \left( \frac{0.025}{0.055} \right) \]
\[ \implies T = 310.33 \]

3. A gas, while expanding under isobaric conditions, does 480 joules of work. The pressure of the gas is 160 kPa, and its initial volume is 0.0015 m³. What is the final volume of the gas?

**Solution**
The work done by a gas under isobaric conditions is \( W = P\Delta V \). Thus,
\[(480) = (160000)(\Delta V) \implies \Delta V = 0.0030 \]
The final volume must be
\[(0.0015) + (0.0030) = 0.0045 \]

4. Beginning with a pressure of 220 kPa and a volume of 0.00634 m³, an ideal monatomic gas (\( \gamma = 5/3 \)) undergoes an adiabatic expansion such that its final pressure is 81.5 kPa. An alternative process leading to the same final state begins with an isochloric cooling to the final pressure, followed by an isobaric expansion to the final volume. How much more work does the gas do in the adiabatic process than the alternative process?

**Solution**
In any adiabatic process, the pressure and volume are related according to
\[ P_1 V_1^\gamma = P_2 V_2^\gamma \]
This allows us to calculate the final volume of the gas:
\[(220000)(0.00634)^{(5/3)} = (81500)(V_2)^{(5/3)} \implies V_2 = 0.011504 \]
Now, the work done in an adiabatic process is given by
\[ W = -\frac{3}{2}nRT\Delta T \]
Which means we really need to know the temperatures involved here. We can use the ideal gas law \( PV = nRT \) to help. At the beginning we have:
\[(220000)(0.00634) = nRT_i \implies nRT_i = 1394.8 \]
And at the end we have:
\[(81500)(0.011504) = nRT_f \implies nRT_f = 937.58 \]
Notice that in the formula for adiabatic work, the quantity we need is the difference between these two:
\[ nR\Delta T = (937.58) - (1394.8) = -457.22 \]
Thus,
\[ W = \left( -\frac{3}{2} \right)(-457.22) = 685.83 \]
Now for the alternate process. The isochoric half means that the volume stays unchanged, so no work is done (although heat is being extracted to reduce the temperature and pressure). The work comes from the isobaric process. Its formula is

$$W = P\Delta V$$

In this case the pressure is at the lower 81.5 kPa and the volume differential is

$$\Delta V = 0.011504 - 0.00634 = 0.0051640$$

So the total work done by the gas in the alternative process is

$$W = (81500)(0.0051640) = 420.87$$

The adiabatic process does more workelvin. The difference is

$$685.83 - 420.87 = 264.96$$

5. The temperature of 2.5 moles of a monatomic ideal gas is 350 K. The internal energy of this gas is doubled by the addition of heat. How much heat is needed when it is added at (a) constant volume and (b) constant pressure?

Solution

(a) The internal energy of the gas is given by \( U = \frac{3}{2}nRT \). Initially, we have

$$U = \frac{3}{2}(2.5)(8.31)(350) = 10907$$

We are told this doubles to \( U = 21814 \), or

$$\Delta U = 10907$$

If the gas is under constant volume, then the amount of work done is zero. So all of the heat input goes into internal energy. Thus, \( Q = 10907 \).

(b) But if the gas is under constant pressure, the gas will expand. So some of the heat will go into work—we will need more heat to get to the same level of internal energy. The work done is given by \( W = nR\Delta T \), so

$$W = (2.5)(8.31)(\Delta T)$$

But what is \( \Delta T \)? For an ideal gas, the temperature is directly related to its internal energy. Since the internal energy doubles, so does its temperature. So, \( \Delta T = 350 \). Therefore,

$$W = (2.5)(8.31)(350) = 7271.3$$

So the gas does 7.2 kJ of work. According to the first law, we have

$$\Delta U = Q - W \implies (10907) = (Q) - (7271.3) \implies Q = 18178$$

6. An engine does 18500 joules of work and rejects 6550 joules of heat into a cold reservoir whose temperature is 285 K. What would be the smallest possible temperature of the hot reservoir?

Solution

The heat input into this engine is

$$Q_h = 18500 + 6550 = 25050$$

So its efficiency is

$$e = \frac{W}{Q_h} = \frac{18500}{25050} = 0.73852$$
The coldest hot reservoir will be obtained using a reversible engine. Its efficiency is given by \( e = 1 - T_c/T_h \). Thus,

\[
0.73852 = 1 - \frac{285}{T_h} \implies T_h = 1090.0
\]

7. A Carnot engine operates between temperatures of 650 and 350 K. To improve the efficiency of the engine, it is decided either to raise the temperature of the hot reservoir by 40 K or to lower the temperature of the cold reservoir by 40 K. Which change gives the greatest improvement? Justify your answer by calculating the efficiency in each case.

Solution

The efficiency is given by \( e = 1 - T_c/T_h \). Thus,

\[
e = 1 - \frac{350}{650} = 0.46154
\]

If we raise the temperature of the hot reservoir we get:

\[
e = 1 - \frac{350}{690} = 0.49275
\]

If we lower the temperature of the cold reservoir we get:

\[
e = 1 - \frac{310}{650} = 0.52308
\]

8. On a cold day, 24500 joules of heat leaks out of a house. The inside temperature is 21 °C, and the outside temperature is -15 °C. What is the increase in the entropy of the universe that this heat loss produces?

Solution

Since entropy does not depend on the way in which we get between states, the important thing to do here is to replace the process with one in which we can do the calculation. This means we want the heat to flow only when the temperature is held constant (isothermal). All other changes must be adiabatic (no heat flow). In this case we can do this by imagining a parallel process which extracts 24500 joules of heat at 21 °C (or 294 K), then reduces the temperature adiabatically, and then adds the heat back at -15 °C (or 258 K).

For the first isothermal process, the entropy change is

\[
\Delta S = \frac{\Delta Q}{T} = \frac{-24500}{294} = -83.333
\]

In the second adiabatic process, the entropy change is zero. For the third isothermal process, the entropy change is

\[
\Delta S = \frac{\Delta Q}{T} = \frac{24500}{258} = 94.961
\]

The total entropy change is the sum of all the parts. Thus,

\[
\Delta S = (-83.333) + (94.961) = 11.628
\]

9. An irreversible engine operates between temperatures of 852 and 314 Kelvin. It absorbs 1285 joules of heat from the hot reservoir and does 264 joules of work. (a) What is the change \( \Delta S \)

Solution

(a) Because 264 joules of work are performed with the input of 1285 joules of heat, there is 1021 joules of heat wasted into the cold reservoir. Entropy does
not depend on the path taken between the two states, so all we need to do is to imagine a process which we can calculate that produces the same end result from the initial conditions. In this case, we can do this in three steps. First, take 1285 joules of heat from the hot reservoir at 852 kelvin. This is an isothermal process so the entropy change is \( \Delta S = \Delta Q / T \). Second, reduce the temperature adiabatically to 314 kelvin to match the cold reservoir. Since no heat flows, the entropy change is zero. Third, push the 1021 joules of heat into the cold reservoir isothermally. This has the net effect of transporting 1021 joules of heat from hot to cold in a way that we can calculate.

For the first isothermal process, the entropy change is

\[
\Delta S = \frac{\Delta Q}{T} = \frac{-1285}{852} = -1.5082
\]

In the second adiabatic process, the entropy change is zero. For the third isothermal process, the entropy change is

\[
\Delta S = \frac{\Delta Q}{T} = \frac{1021}{314} = 3.2516
\]

The total entropy change is the sum of all the parts. Thus,

\[
\Delta S = (-1.5082) + (3.2516) = 1.7434
\]

(b) For an irreversible engine, the entropy change is zero. The efficiency of such an engine is given by \( e = 1 - T_c / T_h \). In our case that is

\[
e = 1 - \frac{314}{852} = 0.63146
\]

Since efficiency is defined as work produced divided by energy input \( e = W / Q_h \) the work done by the reversible engine must be

\[
(0.63146) = \frac{W}{1285} \implies W = 811.42
\]

(c) The difference between the reversible engine and this irreversible one is

\[
811.42 - 264 = 547.42
\]

Notice that this can also be calculated using

\[
W_{lost} = T_c \Delta S = (314)(1.7434) = 547.43
\]

10. Suppose a monatomic ideal gas is contained within a vertical cylinder that is fitted with a movable piston. The piston is frictionless and has a negligible mass. The radius of the piston is 10.0 cm, and the pressure outside the cylinder is 101 kPa. Heat (2093 joules) is removed from the gas. Through what distance does the piston drop?

**Solution**

Since the piston is of negligible mass, the pressure in the gas is only supporting it against atmospheric pressure and no work is done. Since the piston moves, the pressure in the gas is 101 kPa before and after the heat is removed. The gas still obeys the ideal gas law \( PV = nRT \). When the piston moves, the volume of the gas changes. We need to determine the right-side of this equation based on the removal of heat.

Removing heat will decrease the temperature of the gas according to \( \Delta Q = \frac{5}{2} nR \Delta T \) because this is an isobaric process. Thus,

\[
(2093) = \frac{5}{2} (nR \Delta T) \implies nR \Delta T = 837.20
\]
Now we can rewrite the ideal gas law {We can only get away with this because $P$ is constant.} as

$$P \Delta V = nR \Delta T$$

Which allows us to calculate $\Delta V$. Thus,

$$(101000)(\Delta V) = 837.20 \implies \Delta V = 8.2891 \times 10^{-3}$$

Since the cross-sectional area of the piston is

$$A = \pi r^2 = (\pi)(0.100)^2 = 3.1416 \times 10^{-2}$$

The change in length must be

$$\Delta l = \frac{\Delta V}{A} = \frac{8.2891 \times 10^{-3}}{3.1416 \times 10^{-2}} = 0.26385$$
1. A loudspeaker has a circular opening with a radius of 0.0950 meters. The electrical power needed to operate the speaker is 25.0 watts. The average sound intensity at the opening is 17.5 W/m². What percentage of the electrical power is converted by the speaker into sound power?

**Solution**

The key equation here is

\[ I = \frac{P}{A} \]

We are given the sound intensity and the radius (and therefore the area). The area in this case is

\[ A = \pi r^2 = \pi (0.0950)^2 = 0.02835 \]

Thus, the power radiated is

\[ (17.5) = \frac{P}{0.02835} \implies P = 0.4962 \]

However, the speaker is using 25.0 W total. The percent converted into sound power is

\[ \frac{0.4962}{25.0} = 0.0198 \]

2. Light is an electromagnetic wave and travels at a speed of \( 3.00 \times 10^8 \) m/s. The human eye is most sensitive to yellow-green light, which has a wavelength of \( 5.45 \times 10^{-7} \) meters. What is the frequency of this light?

**Solution**

\[ f = \lambda \frac{v}{c} = (5.45 \times 10^{-7}) \frac{3.00 \times 10^8}{5.50 \times 10^{14}} = 9.81 \times 10^{19} \text{ Hz} \]

3. Suppose the linear density of the A string on a violin is \( 7.8 \times 10^{-4} \) kg/m. A wave on the string has a frequency of 440 Hz and a wavelength of 65 cm. What is the tension in the string?

**Solution**

The tension is related to the speed of the wave: \( v = \sqrt{\frac{F}{\mu}} \). In addition, the speed of the wave is related to its frequency and wavelength by \( v = f\lambda \). Thus,

\[ v = (440)(0.65) = 286 \]

Which we can plug into first equation:

\[ (286) = \sqrt{\frac{F}{7.8 \times 10^{-4}}} \implies F = 63.801 \]

4. The mass of a string is 5.0 grams, and it is stretched so that the tension in it is 180 newtons. A transverse wave traveling on this string has a frequency of 260 Hz and a wavelength of 0.60 meters. What is the length of the string?

**Solution**

The speed of the wave on the string is given by the formula \( v = \sqrt{\frac{F}{\mu}} \). In addition, since we know the frequency and wavelength, we can determine this speed in this case:

\[ v = f\lambda = (260)(0.60) = 156 \]

Thus,

\[ (156) = \sqrt{\frac{180}{\mu}} \implies \mu = 7.3964 \times 10^{-3} \]
By definition, the linear density of the string is mass per unit length. In symbols,

$$\mu = \frac{M}{L}$$

We can use this last formula to figure out the length of this string:

$$(7.3964 \times 10^{-3}) = \frac{5.0 \times 10^{-3}}{L} \implies L = 0.67600$$

5. A copper wire, whose cross-sectional area is $1.1 \times 10^{-6}$ m$^2$, has a linear density of $9.8 \times 10^{-3}$ kg/m and is strung between two walls. At the ambient temperature, a transverse wave travels with a speed of 46 m/s on this wire. The coefficient of linear expansion for copper is $17 \times 10^{-6}$ °C$^{-1}$, and Young’s modulus for copper is $1.1 \times 10^{11}$ N/m$^2$. What will be the speed of the wave when the temperature is lowered by 14 °C? Ignore any change in the linear density caused by the change in temperature.

**Solution**

When the temperature is lowered, the wire will contract slightly. This will increase the tension and the propagation speed of the wave. Initially, the tension in the wire can be determined using $v = \sqrt{F/\mu}$. Thus,

$$(46) = \sqrt{(F)/(9.8 \times 10^{-3})} \implies F = 20.737$$

If the wire were free to contract, the amount of contraction would be given by $\Delta L/L_0 = \alpha \Delta T$. In this case,

$$\frac{\Delta L}{L} = (17 \times 10^{-6})(14) = 2.3800 \times 10^{-4}$$

But the wire is not allowed to contract. This additional strain manifests as extra tension in the wire according to $F/A = (Y)(\Delta L/L_0)$. Our data yields:

$$\frac{F}{1.1 \times 10^{-6}} = (1.1 \times 10^{11})(2.3800 \times 10^{-4})$$

$$\implies F = 28.798$$

This is the extra tension introduced by the temperature difference. The total tension is now

$$F = 20.737 + 28.798 = 49.535$$

Therefore the propagation speed is bumped up to

$$v = \sqrt{\frac{49.535}{9.8 \times 10^{-3}}} = 71.096$$

6. Have you ever listened for an approaching train by kneeling next to a railroad track and putting your ear to the rail? Young’s modulus for steel is $2.0 \times 10^{11}$ N/m$^2$, and the density of steel is 7860 kg/m$^3$. On a day when the temperature is 20 °C, how many times greater is the speed of sound in the rail than in the air?

**Solution**

The speed of sound in the steel is given by

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.0 \times 10^{11}}{7860}} = 5044.3$$

But the speed of sound in air at 20 °C is 343 m/s. The ratio of these two is:

$$\frac{5044.3}{343} = 14.707$$
7. When an earthquake occurs, two types of sound waves are generated and travel through the earth. The primary, or P, wave has a speed of about 8000 m/s and the secondary, or S, wave has a speed of about 4500 m/s. A seismograph, located some distance away, records the arrival of the P wave and then, 78 seconds later, records the arrival of the S wave. Assuming that the waves travel in a straight line, how far is the seismograph from the earthquake?

**Solution**

When the problem says that the waves “travel in a straight line”, it means that we are really talking about rays of sound here. Let’s use \( d \) to label the distance the rays travel. Both waves travel at constant velocity so we can use the kinematic formula \( x = vt \) for both waves. For the P wave, we have

\[
(d) = (8000)(t)
\]

And for the S wave we have

\[
(d) = (4500)(t + 78)
\]

The first equation gives us an expression for \( t \):

\[
t = \frac{d}{8000}
\]

Which we can plug into the second to get:

\[
(d) = (4500)\left[\frac{d}{8000} + 78\right]
\]

Or

\[
\frac{d}{4500} = \frac{d}{8000} + 78
\]

We could simply have started here, because this formula simply says that the time it takes for the P wave to travel the distance \( d \) (the left-hand-side of the equation) is equal to the time for the S wave plus 78 seconds. In any case, we solve for \( d \):

\[
(9.7222 \times 10^{-5})(d) = 78 \implies d = 802290
\]

8. A dish of lasagna is being heated in a microwave oven. The effective area of the lasagna that is exposed to the microwaves is 0.022 m\(^2\). The mass of the lasagna is 0.35 kilograms, and its specific heat capacity is 3200 J/(kg\(^\circ\)C). The temperature rises by 72 \(^\circ\)C in 8.0 minutes. What is the intensity of the microwaves in the oven?

**Solution**

The heat required to raise the temperature of the lasagna is given by \( \Delta Q = cm\Delta T \), so

\[
\Delta Q = (3200)(0.35)(72) = 80640
\]

This energy is delivered over 8.0 minutes (or 480 seconds) which corresponds to a power rating of

\[
P = \frac{E}{t} = \frac{80640}{480} = 168.00
\]

Since intensity is power divided by surface area, we have

\[
I = \frac{P}{A} = \frac{168.00}{0.022} = 7636.3
\]

9. A woman stands a distance \( d \) from a loud motor that emits sound uniformly in all directions. The sound intensity at her position is an uncomfortable \( 3.2 \times 10^{-3} \) W/m\(^2\). At a position twice as far from the motor, what are (a) the sound intensity and (b) the sound intensity level relative to the threshold of hearing?

(a) \( 8.0 \times 10^{-4} \) W/m\(^2\)

(b) 89 dB
Solution

(a) The radiation intensity from a point source obeys the formula

\[ I = \frac{P}{4\pi r^2} \]

So if the distance from the source (i.e. \( r \)) doubles, then the intensity will be reduced by a factor of one-fourth. Thus,

\[ I = \left(\frac{1}{4}\right)(3.2 \times 10^{-3}) = 8.0 \times 10^{-4} \]

(b) The sound intensity level is given by

\[ \beta = 10 \log\left(\frac{I}{10^{-12}}\right) \]

Thus,

\[ \beta = 10 \log\left(\frac{8.0 \times 10^{-4}}{10^{-12}}\right) = 89.031 \]

10. When one person shouts at a football game, the sound intensity level at the center of the field is 60.0 dB. When all the people shout together, the intensity level increases to 109 dB. Assuming that each person generates the same sound intensity at the center of the field, how many people are at the game?

Solution

The intensity level \( (\beta) \) is related to the intensity \( (I) \) according to

\[ \beta = 10 \log\left(\frac{I}{10^{-12}}\right) \]

Therefore, the sound intensity from one person is

\[ (60) = (10) \log\left(\frac{I}{10^{-12}}\right) \Rightarrow I = 1.0000 \times 10^{-6} \]

And the sound intensity from all the people is

\[ (109) = (10) \log\left(\frac{I}{10^{-12}}\right) \Rightarrow I = 7.9433 \times 10^{-2} \]

Since intensity is additive the number of people must be:

\[ N = \frac{7.9433 \times 10^{-2}}{1.0000 \times 10^{-6}} = 79433 \]
1. On a cello, the string with the largest linear density (0.0156 kg/m) is the C string. This string produces a fundamental frequency of 65.4 Hz and has a length of 0.800 meters between the two fixed ends. Find the tension in the string.

Solution

The tension in the string is involved in the formula \( v = \sqrt{F/\mu} \). We know \( \mu \), but we need to calculate \( v \). We can get this from the fundamental frequency. Its formula is

\[ f = (2n) \frac{v}{4L} \quad n = 1, 2, 3, ... \]

We can use this formula to extract \( v \). Thus,

\[ (65.4) = (2) \frac{v}{(4)(0.800)} \implies v = 104.64 \]

So, the tension is

\[ (104.64) = \sqrt{\frac{F}{0.0156}} \implies F = 170.81 \]

2. Two speakers, one directly behind the other, are each generating a 245-Hz sound wave. What is the smallest separation distance between the speakers that will produce destructive interference at a listener standing in front of them? The speed of sound is 343 m/s.

Solution

The path-length difference between the speakers and the listener must be half a wavelength—this will generate the destructive interference we seek. So, the question becomes: what is the wavelength of this sound wave? The equation to use is \( v = f\lambda \). Thus,

\[ (343) = (245)(\lambda) \implies \lambda = 1.4000 \]

Half of this is 0.700 meters. So this is the first place to experience the destructive interference.

3. Suppose that the two speakers in Figure 4 are separated by 2.50 meters and are vibrating exactly out of phase at a frequency of 429 Hz. The speed of sound is 343 m/s. Does the observer at C observe constructive or destructive interference when his distance from speaker B is (a) 1.15 meters and (b) 2.00 meters?

Figure 4: Problem 17.8
Solution

(a) Since the two speakers are exactly out of phase, there is already one-half of a wavelength difference between them. The wavelength of this sound is given by
\[ v = f\lambda \]
\[ (343) = (429)(\lambda) \implies \lambda = 0.79953 \]
The distance between speaker C and A is
\[ l = \sqrt{(1.15)^2 + (2.50)^2} = 2.7518 \]
The path-length difference between the two speakers is
\[ \Delta l = 2.7518 - 1.15 = 1.6018 \]
The number of wavelengths in this path-length difference is
\[ N = \frac{1.6018}{0.79953} = 2.0034 \]
Since there is already a half-wavelength difference between the speakers, this will lead to destructive interference.

(b) This logic is the same. The distance between speaker C and A is
\[ l = \sqrt{(2.00)^2 + (2.50)^2} = 3.2016 \]
The path-length difference between the two speakers is
\[ \Delta l = 3.2016 - 2.00 = 1.2016 \]
The number of wavelengths in this path-length difference is
\[ N = \frac{1.2016}{0.79953} = 1.5028 \]
The extra half-wavelength of difference between the speakers makes this a whole integer. Therefore there will be constructive interference at this point.

4. Speakers A and B are vibrating in phase. They are directly facing each other, are 7.80 meters apart, and are each playing a 73.0-Hz tone. The speed of sound is 343 m/s. On the line between the speakers there are three points where constructive interference occurs. What are the distances of these three points from speaker A?

Solution

We really want the wavelength of the sound in order to analyze the interference pattern. This we can extract from \( v = f\lambda \). Thus,
\[ (343) = (73.0)(\lambda) = \lambda = 4.6986 \]
Constructive interference will occur when the path-length difference from the two sources is an integer number of wavelengths. Let’s give the distance from speaker A the symbol \( d \). Then the distance from speaker B must be \( 7.80 - d \). The path-length difference is:
\[ \Delta l = (d) - (7.80 - d) = 2d - 7.80 \]
As mentioned earlier, this has to be equal to an integer number of wavelengths to create constructive interference. Thus,
\[ 2d - 7.80 = (n)(4.6986) \]
If we set \( n = 0 \), we have
\[ 2d - 7.80 = (0)(4.6986) \implies d = 3.9000 \]
If we set \( n = 1 \), we have
\[ 2d - 7.80 = (1)(4.6986) \implies d = 6.2493 \]
If we set \( n = -1 \), we have
\[ 2d - 7.80 = (-1)(4.6986) \implies d = 1.5507 \]
There are more points of constructive interference with \( |n| > 1 \), but these three are the only ones that fall in between the two speakers.

5. The entrance to a large lecture room consists of two side-by-side doors, one hinged on the left and the other hinged on the right. Each door is 0.700 meters wide. Sound of frequency 607 Hz is coming through the entrance from within the room. The speed of sound is 343 m/s. What is the diffraction angle of the sound after it passes through the doorway when (a) one door is open and (b) both doors are open?

**Solution**

(a) The frequency of the sound is given by \( v = f\lambda \). Thus,
\[ (343) = (607)(\lambda) \implies \lambda = 0.56507 \]
The diffraction angle is given by the formula \( \sin \theta = \lambda/D \). With one door open, the “slit distance” \( D \) is 0.700 meters. Thus,
\[ \sin \theta = \frac{0.56507}{0.700} \implies \theta = 53.828^\circ \]
(b) With both doors open, we have \( D = 1.400 \) meters. Thus,
\[ \sin \theta = \frac{0.56507}{1.400} \implies \theta = 23.805^\circ \]

6. A 3.00-kHz tone is being produced by a speaker with a diameter of 0.175 meters. The air temperature changes from 0 to 29 °C. Assuming air to be an ideal gas, find the change in the diffraction angle.

**Solution**

The formula for the diffraction angle is \( \sin \theta = (1.22)(\lambda/D) \). The temperature change affects the speed of sound which will affect the wavelength associated with the 3.00-kHz tone (according to \( v = f\lambda \)). The speed of sound at 0 °C is 331 m/s. The relationship between the speed of sound and temperature is \( v = \sqrt{kT/m} \).

If we take this formula, apply it to both temperatures and divide, most of these constants cancel. Thus,
\[ \frac{v_1}{v_2} = \frac{\sqrt{\gamma kT_1/m}}{\sqrt{\gamma kT_2/m}} = \sqrt{\frac{T_1}{T_2}} \]
Thus,
\[ \frac{v}{331} = \sqrt{\frac{29 + 273}{0 + 273}} \implies v = 348.14 \]
So, at 0 °C the wavelength is
\[ (331) = (3000)(\lambda) \implies \lambda = 0.11033 \]
With a diffraction angle of
\[ \sin \theta = (1.22)\frac{0.11033}{0.175} \implies \theta = 50.280^\circ \]
And at 29 °C the wavelength is
\[ (348.14) = (3000)(\lambda) \implies \lambda = 0.11605 \]
So its diffraction angle will be

\[
\sin \theta = (1.22) \frac{0.11605}{0.175} \implies \theta = 53.999^\circ
\]

Therefore, the change in diffraction angle is

\[
\Delta \theta = 53.999^\circ - 50.280^\circ = 3.719^\circ
\]

7. Two strings have different lengths and linear densities, as Figure 5 shows. They are joined together and stretched so that the tension in each string is 190.0 newtons. The free ends of the joined string are fixed in place. Find the lowest frequency that permits standing waves in both strings with a node at the junction. The standing wave pattern in each string may have a different number of antinodes.

![Figure 5: Problem 17.36](image)

**Solution**

We can calculate the speed of propagation in each of these strings using \(v = \sqrt{F/\mu}\). For the left string we have

\[
v_L = \sqrt{\frac{190.0}{0.0600}} = 56.273
\]

And on the right we have

\[
v_R = \sqrt{\frac{190.0}{0.0150}} = 112.55
\]

In effect, we have two strings which are fixed at both ends. The frequency of these standing waves follow the formula \(f = (2n)(v/4L)\). Thus,

\[
f_L = (2n_L) \frac{56.273}{(4)(3.75)} = (7.5031)(n_L)
\]

and

\[
f_R = (2n_R) \frac{112.55}{(4)(1.25)} = (45.020)(n_R)
\]

These two frequencies must match. Thus

\[
(7.5031)(n_L) = (45.020)(n_R)
\]

Or,

\[
(n_L) = (6.0002)(n_R)
\]

The lowest frequency will occur if \(n_R = 1\) and \(n_L = 6\). We can take either one and plug it into the frequency formula:

\[
f_L = (7.5031)(6) = 45.012
\]

Or

\[
f_R = (45.020)(1) = 45.020
\]
8. Sound enters the ear, travels through the auditory canal, and reaches the eardrum. The auditory canal is approximately a tube open at only one end. The other end is closed by the eardrum. A typical length for the auditory canal in an adult is about 2.9 cm. The speed of sound is 343 m/s. What is the fundamental frequency of the canal? (Interestingly, the fundamental frequency is in the frequency range where human hearing is most sensitive.)

Solution

Since we are dealing with a tube closed at one end, the frequency of the standing waves is given by the formula

\[ f = \frac{(2n-1)v}{4L} \quad n = 1, 2, 3, ... \]

with the fundamental frequency associated with \( n = 1 \). Thus,

\[ f = \frac{343}{(4)(0.029)} = 2956.9 \]

9. A tube, open at only one end, is cut into two shorter (nonequal) lengths. The piece that is open at both ends has a fundamental frequency of 425 Hz, while the piece open only at one end has a fundamental frequency of 675 Hz. What is the fundamental frequency of the original tube?

Solution

The equation for the frequencies supported in a tube open at both ends is

\[ f_n = \frac{(2n)v}{4L} \quad n = 1, 2, 3, ... \]

The equation for the frequencies supported in a tube open at only one ends is

\[ f_n = \frac{(2n-1)v}{4L} \quad n = 1, 2, 3, ... \]

The fundamental frequency implies that \( n = 1 \). So, for the first half-tube we have:

\[ (425) = (2) \frac{v}{4L_1} \implies L_1 = (1.1765 \times 10^{-3})(v) \]

and for the second half-tube we have:

\[ (675) = (1) \frac{v}{4L_2} \implies L_2 = (3.7037 \times 10^{-4})(v) \]

We don't know \( v \), but we do know that it is the same for all the tubes.

Since the original tube is only open at one end, its fundamental frequency is given by

\[ f = \frac{v}{4L_0} \]

where \( L_0 = L_1 + L_2 \). Or,

\[ L_0 = (1.1765 \times 10^{-3})(v) + (3.7037 \times 10^{-4})(v) = (1.5469 \times 10^{-3})(v) \]

Therefore,

\[ f = \frac{v}{(4)(1.5469 \times 10^{-3})(v)} = 161.62 \]

10. A pipe open only at one end has a fundamental frequency of 256 Hz. A second pipe, initially identical to the first pipe, is shortened by cutting off a portion of the open end. Now, when both pipes vibrate at their fundamental frequencies, a beat frequency of 12 Hz is heard. How many centimeters were cut off the end of the second pipe? The speed of sound is 343 m/s.

\[ 0.029 \text{ cm} \]

1.5 centimeters
Solution

Since the second pipe is shorter, the fundamental frequency is larger. The beat frequency is the difference between the fundamental frequencies. So the fundamental frequency of the second pipe is 268 Hz.

The pipes are closed at one end, so the fundamental frequencies are given by

\[ f = \frac{v}{4L} \]

For the first pipe we have

\[ 256 = \frac{343}{4L_1} \Rightarrow L_1 = 0.33496 \]

and for the second pipe we have

\[ 268 = \frac{343}{4L_2} \Rightarrow L_2 = 0.31996 \]

The difference between these two is what we are looking for:

\[ 0.33496 - 0.31996 = 0.01500 \]
1. A beam of sunlight encounters a plate of crown glass at a 45.00° angle of incidence. Using the data in Figure 6, find the angle between the violet ray and the red ray in the glass.

<table>
<thead>
<tr>
<th>Color</th>
<th>Vacuum Wavelength (nm)</th>
<th>Index of Refraction, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>660</td>
<td>1.520</td>
</tr>
<tr>
<td>Orange</td>
<td>610</td>
<td>1.522</td>
</tr>
<tr>
<td>Yellow</td>
<td>580</td>
<td>1.523</td>
</tr>
<tr>
<td>Green</td>
<td>550</td>
<td>1.526</td>
</tr>
<tr>
<td>Blue</td>
<td>470</td>
<td>1.531</td>
</tr>
<tr>
<td>Violet</td>
<td>410</td>
<td>1.538</td>
</tr>
</tbody>
</table>

*Approximate

Solution

For the red ray, $n = 1.520$. According to Snell’s law, the angle of refraction inside the glass is

$$\sin(45.00°) = (1.520) \sin \theta \Rightarrow \theta = 27.723°$$

And for the violet ray, $n = 1.538$. So,

$$\sin(45.00°) = (1.538) \sin \theta \Rightarrow \theta = 27.371°$$

The difference is

$$27.723° - 27.371° = 0.352°$$

2. You are trying to photograph a bird sitting on a tree branch, but a tall hedge is blocking your view. However, as Figure 7 shows, a plane mirror reflects light from the bird into your camera. For what distance must you set the focus of the camera lens in order to snap a sharp picture of the bird’s image?

Solution

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$$3.7 m$$
The image of the bird is located into the mirror at the same distance the object is from the mirror, in this case 2.1 meters. This means the horizontal distance from the camera to the image is the 3.7 meters from the camera to the mirror and an additional 2.1 meters to the image, or 5.8 meters total. The vertical distance is still 4.3 meters. With the image on the other side, a right triangle is formed. By the Pythagorean theorem, the distance is
\[ d = \sqrt{(5.8)^2 + (4.3)^2} = 7.2201 \]

3. A small statue has a height of 3.5 cm and is placed in front of a concave mirror. The image of the statue is inverted, 1.5 cm tall, and is located 13 cm in front of the mirror. Find the focal length of the mirror.

Solution

By definition, the magnification is \( m = h_i/h_o \). In this case, since the image is inverted, the image height is negative. Thus,
\[ m = \frac{-1.5}{3.5} = -0.42857 \]

The magnification tells us something about the image and object distances according to \( m = -d_i/d_o \). Thus,
\[ (-0.42857) = -\frac{d_i}{d_o} \]

Since we know the image distance, we can calculate the object distance:
\[ (-0.42857) = -\frac{13}{d_o} \implies d_o = 30.333 \]

We can now use the lens equation \( \left(\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}\right) \) to calculate the focal length:
\[ \frac{1}{30.333} + \frac{1}{13} = \frac{1}{f} \implies f = 9.1000 \]

4. The outside mirror on the passenger side of a car is convex and has a focal length of -7.0 meters. Relative to this mirror, a truck traveling in the rear has an object distance of 11 meters. Find (a) the image distance of the truck and (b) the magnification of the mirror.

Solution

(a) The lens equation is
\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]

Thus,
\[ \frac{1}{11} + \frac{1}{d_i} = \frac{1}{-7} \implies d_i = -4.2778 \]

(b) The magnification can be derived from
\[ m = -\frac{d_i}{d_o} \]

thus,
\[ m = -\frac{-4.2778}{11} = 0.38889 \]

5. A spherical mirror is polished on both sides. When the convex side is used as a mirror, the magnification is 0.25. What is the magnification when the concave side is used as a mirror, the object remaining the same distance from the mirror?
Solution

The magnification equation states \( m = -d_i/d_o \), so
\[
d_i = -(0.25)(d_o)
\]
The lens equation states \( 1/d_o + 1/d_i = 1/f \), so
\[
\frac{1}{d_o} + \frac{1}{-(0.25)(d_o)} = \frac{1}{f}
\Rightarrow \frac{1}{d_o} - \frac{4}{d_o} = \frac{1}{f}
\Rightarrow \frac{-3}{d_o} = \frac{1}{f}
\Rightarrow f = -d_o/3
\]

Now if we flip the mirror the new focal length is simply the original with the sign flipped. Thus,
\[ f = d_o/3 \]
We can plug this into another lens equation to get
\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{3}{d_o}
\Rightarrow \frac{1}{d_i} = \frac{2}{d_o}
\Rightarrow d_i = d_o/2
\]
Therefore the magnification must be
\[
m = - \frac{d_i}{d_o} = -\frac{d_o/2}{d_o} = -0.5000
\]

6. Figure 8 shows a top view of a square room. One wall is missing, and the wall on the right is a mirror. From point \( P \) in the center of the open side, a laser is pointed at the mirrored wall. At what angle of incidence must the light strike the right-hand wall so that, after being reflected, the light hits the left corner of the back wall?

\[
\tan \theta = \frac{L}{(1.5)(L)} = 0.66667 \Rightarrow \theta = 33.690^\circ
\]

Figure 8: Problem 25.47

Solution

Let the length of each side of the room be \( L \). The trick is to find the image of this corner in beyond the mirror. It is simply on the other side one wall-length inside the mirror. The laser must point at this image. The laser beam will run along the hypotenuse of a right triangle with the image of the far wall as the opposite side (with length \( L \)) and the adjacent side is from the laser to the image (therefore a length \( \frac{1}{2}L + L \)). The angle we are looking for is obtained by the tangent function:
\[
\tan \theta = \frac{L}{(1.5)(L)} = 0.66667 \Rightarrow \theta = 33.690^\circ
\]
7. A spotlight on a boat is 2.5 meters above the water, and the light strikes the water at a point that is 8.0 meters horizontally displaced from the spotlight (see Figure 9). The depth of the water is 4.0 meters. Determine the distance \( d \), which locates the point where the light strikes the bottom.

![Figure 9: Problem 26.12](image)

**Solution**

The key here is to apply Snell’s law at the point of refraction. The incident angle is the complement to the acute angle formed by the light ray and the surface of the water. The tangent of this acute angle is related to the distances in the diagram:

\[
\tan \theta = \frac{2.5}{8.0} \implies \theta = 17.354^\circ
\]

So the angle of incidence is

\[
\theta_1 = 90 - 17.354 = 72.646
\]

Snell’s law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \), will tell us the refracted angle:

\[
(1) \sin(72.646^\circ) = (1.33 \sin \theta_2 \implies \theta_2 = 45.861^\circ
\]

This angle is also related to the distances in the drawing. In this case, the adjacent side is the 4.00 meter height and the opposite side is what we want to know. Thus,

\[
\tan(45.861^\circ) = \frac{x}{4.00} \implies x = 4.1221
\]

Of course, we must add the 8.00 meters to get the full length \( d \):

\[
d = 4.1221 + 8.00 = 12.122
\]

8. A diverging lens has a focal length of -25 cm. (a) Find the image distance when an object is placed 38 cm from the lens (b) Is the image real or virtual?

**Solution**

(a) The lens equation is \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \), so

\[
\frac{1}{38} + \frac{1}{d_i} = \frac{1}{-25} \implies d_i = -15.079
\]

(b) Since the image distance is negative, the image is to the left side of the lens. Since the light rays are actually on the right side of the lens, the image is virtual.

9. A camper is trying to start a fire by focusing sunlight onto a piece of paper. The diameter of the sun is \( 1.39 \times 10^9 \) meters, and its mean distance from the earth is \( 1.50 \times 10^{11} \) meters. The camper is using a converging lens whose focal length is 10.0 cm. (a) What is the area of the sun’s image on the paper? (b) If 0.530 watts of sunlight pass through the lens, what is the intensity of the sunlight at the paper?

**Solution**

(a) \( 6.74 \times 10^{-7} \) m²

(b) \( 7.86 \times 10^5 \) W/m²
(a) In this case, the object distance is practically infinite. This means that the image distance is equal to the focal length of the lens. The magnification of the sun would therefore be
\[ m = \frac{-d_i}{d_o} = -\frac{0.100}{1.50 \times 10^{11}} = 6.6667 \times 10^{-13} \]

So the image diameter must be
\[ (1.39 \times 10^9)(6.6667 \times 10^{-13}) = 9.2667 \times 10^{-4} \]

But we are asked for the area which is given by \( A = \pi r^2 \). Thus,
\[ A = (\pi)(4.6333 \times 10^{-4})^2 = 6.7443 \times 10^{-7} \]

(b) The intensity is defined as \( I = \frac{P}{A} \), so
\[ I = \frac{0.530}{6.7443 \times 10^{-7}} = 7.8585 \times 10^5 \]

10. A converging lens (\( f = 25.0 \) cm) is used to project an image of an object onto a screen. The object and the screen are 125 cm apart, and between them the lens can be placed at either of two locations. Find the two object distances.

Solution

Since the distance between the object and the screen is constant, we have
\[ d_o + d_i = 125 \]

In addition, we have the lens equation:
\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25.0} \]

Mathematically, what we have is two equations and two unknowns. We can use the first to substitute out the \( d_i \) in the second to yield:
\[ \frac{1}{d_o} + \frac{1}{125 - d_o} = 0.0400 \]

We need common denominators, so
\[ \frac{125 - d_o}{(d_o)(125 - d_o)} + \frac{d_o}{(d_o)(125 - d_o)} = 0.0400 \]
and add:
\[ \frac{125}{(d_o)(125 - d_o)} = 0.0400 \]
then cross-multiply:
\[ 125 = (0.0400)(d_o)(125 - d_o) \]
and simplify:
\[ d_o^2 - 125d_o + 3125 = 0 \]

This quadratic equation has the solutions:
\[ d_o = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-(-125) \pm \sqrt{(-125)^2 - (4)(1)(3125)}}{(2)(1)} \]
\[ = \frac{125 \pm 55.902}{2} \]
\[ = 90.451 \text{ or } 34.549 \text{ cm} \]
1. A sheet that is made of plastic ($n = 1.60$) covers one slit of a double slit (see Figure 10). When the double slit is illuminated by monochromatic light (wavelength in vacuum = 586 nm), the center of the screen appears dark rather than bright. What is the minimum thickness of the plastic?

![Figure 10: Problem 27.09](image)

**Solution**

Normally the double slit experiment is designed such that the two slits are producing light that is exactly in phase with one another. This is typically done by using the same light source for each slit, but the essential point is that the two slit sources are in phase. In this problem, constructive interference is replaced with destructive interference. This is accomplished by causing the two slit sources to be exactly out of phase with one another. In other words, the plastic needs to change the wavelength just so that an additional half-wavelength is added. This way when the light leaves the plastic it is out of phase.

So, we need to find the particular distance which has one extra half-wavelength when the light is going through the plastic rather than the vacuum. Let diagram calls this distance $t$. The number of wavelengths that are in this distance are:

$$N_{\text{vacuum}} = \frac{t}{5.86 \times 10^{-7}}$$

So, what is the wavelength of the light when it is in the plastic? It is

$$\lambda_{\text{plastic}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{plastic}}}$$

Remember that the index of refraction is related to the speed of the wave: $v = c/n$. When the wave passes through the vacuum-plastic interface, the frequency stays the same, but the speed changes. By $v = f\lambda$, the wavelength must get smaller also. In our case, we have

$$\lambda_{\text{plastic}} = \frac{5.86 \times 10^{-7}}{1.60} = 3.6625 \times 10^{-7}$$

With this wavelength, the number of wavelengths in the distance $t$ are

$$N_{\text{plastic}} = \frac{t}{3.6625 \times 10^{-7}}$$
Because of the reasoning above, these numbers need to differ by one-half of a wavelength. Thus, we must have

\[ N_{\text{plastic}} = N_{\text{vacuum}} + \frac{1}{2} \]

After we plug all this in, we can solve for \( t \):

\[
\frac{t}{3.6625 \times 10^{-7}} = \frac{t}{5.86 \times 10^{-7}} + \frac{1}{2}
\]

\[
\Rightarrow (2.7304 \times 10^7)(t) = (1.7065 \times 10^7)(t) + \frac{1}{2}
\]

\[
\Rightarrow (1.0239 \times 10^7)(t) = \frac{1}{2}
\]

\[
\Rightarrow t = 4.8833 \times 10^{-7}
\]

2. The transmitting antenna for a radio station is 7.00 km from your house. The frequency of the electromagnetic wave broadcast by this station is 536 kHz. The station builds a second transmitting antenna that broadcasts an identical electromagnetic wave in phase with the original one. The new antenna is 8.12 km from your house. Does constructive or destructive interference occur at the receiving antenna of your radio? Show your calculations.

**Solution**

In these interference calculations, the important quantity is the wavelength of the wave. Since the speed of light is \( c = 3.00 \times 10^8 \) m/s, we can figure out the wavelength based on the frequency using \( v = f\lambda \):

\[ 3.00 \times 10^8 = (5.36 \times 10^5)(\lambda) \Rightarrow \lambda = 559.70 \]

The question of interference centers on the question of how many wavelengths fit in the path-length difference from the two sources. In this case, that difference is

\[ \Delta l = 8120 - 7000 = 1120 \]

The number of wavelengths is

\[ N = \frac{1120}{559.70} = 2.0011 \]

Since this is nearly a whole integer, there will be constructive interference.

3. A single slit has a width of \( 2.1 \times 10^{-6} \) and is used to form a diffraction pattern. Find the angle that locates the second dark fringe when the wavelength of the light is (a) 430 nm and (b) 660 nm.

**Solution**

(a) The dark fringes from single slit diffraction are located via the formula

\[ \sin \theta = \frac{m \lambda}{W} \]

Therefore,

\[ \sin \theta = (2) \frac{4.30 \times 10^{-7}}{2.1 \times 10^{-6}} \Rightarrow \theta = 24.175^o \]

(b) Similarly,

\[ \sin \theta = (2) \frac{6.60 \times 10^{-7}}{2.1 \times 10^{-6}} \Rightarrow \theta = 38.945^o \]

4. A flat screen is located 0.60 meters away from a single slit. Light with a wavelength of 510 nm (in vacuum) shines through the slit and produces a diffraction pattern. The width of the central bright fringe on the screen is 0.050 meters. What is the width of the slit?
Solution

The width of the central bright fringe is defined by the location of the dark fringes on either side. Thus, the distance to the first dark fringe is half the width of the central bright fringe: 0.025 meters. The formula for the location of the dark fringes is

\[ \sin \theta = \frac{m \lambda}{W} \]

The \( \theta \) is in a right triangle. The opposite side is on the screen and the adjacent side is the from the slit to the screen. Thus,

\[ \tan \theta = \frac{0.025}{0.60} \implies \theta = 2.3859^\circ \]

Plugging this in, we have

\[ \sin 2.3859^\circ = \frac{(1)(5.10 \times 10^{-7})}{W} \implies W = 1.225 \times 10^{-5} \]

5. In a single-slit diffraction pattern on a flat screen, the central bright fringe is 1.2 cm wide when the slit width is \( 3.2 \times 10^{-5} \) meters. When the slit is replaced by a second slit, the wavelength of the light and the distance to the screen remaining unchanged, the central bright fringe broadens to a width of 1.9 cm. What is the width of the second slit? It may be assumed that \( \theta \) is so small that \( \sin \theta = \tan \theta \).

Solution

The diffraction angle is given by \( \sin \theta = \frac{\lambda}{W} \). Thus,

\[ \sin \theta = \frac{\lambda}{3.2 \times 10^{-5}} \]

The angle in this problem is forming a right triangle whose opposite side is the fringe distance and the adjacent side is the distance between the wall and the screen. So in the first case we have:

\[ \tan \theta = \frac{0.012}{d} \]

Since we are allowed to assume \( \sin \theta = \tan \theta \), we can combine these two equations and cross-multiply:

\[ \frac{\lambda}{3.2 \times 10^{-5}} = \frac{0.012}{d} \implies \lambda d = 3.8400 \times 10^{-7} \]

Remember that both the wavelength and the distance to the screen remain unchanged, so this is true in the second situation also. But in the second situation we don’t know the slit width \( W \). So the combined formula becomes

\[ \frac{\lambda}{W} = \frac{0.019}{d} \]

If we solve for \( W \), we get

\[ W = \frac{\lambda d}{0.019} \]

Or,

\[ W = \frac{3.8400 \times 10^{-7}}{0.019} = 2.0211 \times 10^{-5} \]

6. Two stars are \( 3.7 \times 10^{11} \) meters apart and are equally distant from the earth. A telescope has an objective lens with a diameter of 1.02 meters and just detects these stars as separate objects. Assume that light of wavelength 550 nm is being observed. Also assume that diffraction effects, rather than atmospheric turbulence, limit the resolving power of the telescope. Find the maximum distance that these stars could be from the earth.

\[ 2.0 \times 10^{-5} \text{ meters} \]

\[ 5.6 \times 10^{17} \text{ meters} \]
Solution

Rayleigh’s criterion for resolution is that the central peak of the diffraction patterns cannot overlap. The diffraction pattern is governed by

$$\sin \theta = 1.22 \frac{\lambda}{D}$$

Thus,

$$\sin \theta = (1.22) \frac{5.50 \times 10^{-7}}{1.02} \implies \theta = 3.7692 \times 10^{-5} \text{ degrees}$$

Remember this is the total diffraction angle from a circular aperture. We really want half of this angle in our analysis.

The length from one star to the midpoint between them is the opposite side of a large right triangle involving this angle. The adjacent side is the distance from the earth to the stars. Thus,

$$\tan(1.8846 \times 10^{-5} \text{ degrees}) = \frac{(\frac{1}{2})(3.7 \times 10^{11})}{x} \implies x = 5.6244 \times 10^{17}$$

7. A spotlight sends red light (wavelength of 694.3 nm) to the moon. At the surface of the moon, which is 3.77 x 10^8 meters away, the light strikes a reflector left there by astronauts. The reflected light returns to the earth, where it is detected. When it leaves the spotlight, the circular beam of light has a diameter of about 0.20 meters, and diffraction causes the beam to spread as the light travels to the moon. In effect, the first circular dark fringe in the diffraction pattern defines the size of the central bright spot on the moon. Determine the diameter (not the radius) of the central bright spot on the moon.

Solution

The diffraction angle for the red light is given by $$\sin \theta = (1.22)(\lambda/D)$$ since the aperture is circular. Thus,

$$\sin \theta = (1.22) \frac{6.943 \times 10^{-7}}{0.20} \implies \theta = 2.4266 \times 10^{-4} \text{ degrees}$$

The edge of the beam effectively makes a right triangle with the opposite side being the radius of the spot on the moon. The adjacent side is the distance to the moon. So,

$$\tan(2.4266 \times 10^{-4} \text{ degrees}) = \frac{r}{3.77 \times 10^{8}} \implies r = 1596.7$$

Since we are asked for the diameter we need to double this length:

$$D = 2r = (2)(1596.7) = 3193.4$$

8. For a wavelength of 420 nm, a diffraction grating produces a bright fringe at an angle of 26°. For an unknown wavelength, the same grating produces a bright fringe at an angle of 41°. In both cases the bright fringes are of the same order $$m$$. What is the unknown wavelength?

Solution

The equation governing the diffraction pattern is

$$\sin \theta = m \frac{\lambda}{d}$$

As is often the case when we are given two situations with many things the same, it is easy to take this equation and divide it by itself. Thus,

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{m \lambda_1/d}{m \lambda_2/d}$$
Since the \( m \) and the \( d \) is the same, they cancel. Thus,
\[
\frac{\sin \theta_2}{\sin \theta_1} = \frac{\lambda_2}{\lambda_1}
\]

We can plug in our data to yield
\[
\frac{\sin 41^\circ}{\sin 26^\circ} = \frac{\lambda_2}{420} \implies \lambda_2 = 628.57
\]

9. A diffraction grating has 2604 lines per centimeter, and it produces a principal maximum at 30.0°. The grating is used with light that contains all wavelengths between 410 and 660 nm. What is (are) the wavelength(s) of the incident light that could have produced this maximum?

**Solution**

The principal maxima of a diffraction grating is given by the formula \( \sin \theta = m\lambda/d \) where \( d \) is the separation between the slits. Since we are told there are 2604 slits per centimeter, each pair of slits is separated by \( 1/2604 \)th of a centimeter. Thus,
\[
d = \frac{0.01}{2604} = 3.8402 \times 10^{-6}
\]

Plugging our data into the main formula yields:
\[
\sin 30^\circ = \left( m \right) \frac{\lambda}{3.8402 \times 10^{-6}} \implies \lambda = \frac{1.9201 \times 10^{-6}}{m}
\]

This is 1920 nm divided by \( m \). We just need to run down the integers to see which wavelengths fit in the range 410 to 660 nm. Thus,
\[
m = 1 : \lambda = 1920 \text{ nm} \\
m = 2 : \lambda = 960 \text{ nm} \\
m = 3 : \lambda = 640 \text{ nm} \\
m = 4 : \lambda = 480 \text{ nm} \\
m = 5 : \lambda = 384 \text{ nm}
\]

10. The distance between adjacent slits of a certain diffraction grating is \( 1.250 \times 10^{-5} \) meters. The grating is illuminated by monochromatic light with a wavelength of 656.0 nm, and is then heated so that its temperature increases by 100.0 °C. Determine the change in the angle of the seventh-order principal maximum that occurs as a result of the thermal expansion of the grating. The coefficient of linear expansion for the diffraction grating is \( 1.30 \times 10^{-4}/\text{°C} \). Be sure to include the proper algebraic sign with your answer: plus if the angle increases, negative if the angle decreases.

**Solution**

The diffraction angle is given by \( \sin \theta = m\lambda/d \). Before the temperature change, the diffraction angle is
\[
\sin \theta = (7) \frac{6.56 \times 10^{-7}}{1.250 \times 10^{-5}} \implies \theta = 21.553^\circ
\]

The increase in temperature will increase the slit distance according to \( (\Delta L/L_0) = \alpha \Delta T \). Thus,
\[
\frac{\Delta L}{1.250 \times 10^{-5}} = (1.30 \times 10^{-4})(100) \implies \Delta L = 1.6250 \times 10^{-7}
\]

So the new slit distance is
\[
L = (1.250 \times 10^{-5}) + (1.6250 \times 10^{-7}) = 1.2663 \times 10^{-5}
\]
Therefore the new diffraction angle is

\[
\sin \theta = (7) \frac{6.56 \times 10^{-7}}{1.2663 \times 10^{-5}} \implies \theta = 21.263^\circ
\]

Which corresponds to a change of

\[
\Delta \theta = 21.263^\circ - 21.553^\circ = -0.290^\circ
\]