Applications of Systems of Linear Equations

Example 1

You are probably curious as to where you would ever use systems of equations. Systems of equations can be used in business for break-even points. Let's say that you are going to manufacture a new product. The product that you are going to manufacture costs you $6 per item to make and you also have $400 of fixed monthly costs to pay for the rent of the garage you will make and store your new product in. If we let y = cost and x = the number of items that you will manufacture per month, an equation that models the cost of making your product is $y = 6x + 400$. Now let's say that you are able to sell each of the items you are making for $10. If we let y also equal the amount of money generated by your sales (revenue) and x = the number of items sold, then you have a revenue equation of $y = 10x$. When costs and revenue are equal, you will break even. So if we solve the system of equations, we will know how many items we have to make and sell to break even and how much money we will collect from sales and how much will go out in expenses at the break even point.

$y = 6x + 400$

$y = 10x$ — Substitution would work nicely for solving this type of system since the y variable is already solved for.

$\mathbf{6x + 400} = 10x$ — Let's subtract $6x$ from both sides to get our variables to one side.

$6x \mathbf{- 6x} + 400 = 10x \mathbf{- 6x}$

$400 \mathbf{= 4x}$

$x = 100$ — So we need to make and sell 100 items to break even. Now let's find out how much money will be changing hands at the break even point by substituting 100 in for the x in the second equation.

$y = 10(100) = \$1000$ — $1000 will be generated by sales and $1000 will be the expenses at the break-even point.

If we sell and make more than 100 items, then we will make a profit. If we sell and make less than 100 items we will be losing money.

Example 2

Another situation that lends itself to this type of a problem deals with interest in two separate accounts. Let's say that you have two separate accounts that earn annual simple interest at rates of 5% and 10%. You invested a total of $20,000 into the accounts, but you cannot remember how much you have in each. Your annual statement just comes with the total interest earned of $1450 and you would like to know how much you invested into each account.

It sounds complicated, doesn't it? The trick to solving a problem like this is to find out what you have totals for and what you do not know. We have totals for dollars invested and interest earned, so we will probably need to have an equation for each area. We do not know how much is in each account, so let

x = the amount of dollars in the account that earns 5% interest

y = the amount of dollars in the account that earns 10% interest

We need to make an equation for total dollars that equals the $20,000 and an equation for total interest earned that equals $1450. Since the sum of the two accounts is $20,000, our total dollar equation is:

$x + y = 20000$

Since the interest for each account can be found by multiplying the interest rate as a decimal by the amount of money in the account, the interest from the 5% account is $.05x$ and the interest in the 10% account is $.10y$. The sum of these two amounts is $1450, so the total interest equation is:

$.05x + .10y = 1450$

This makes our system of equations:

$x + y = 20000$

$.05x + .10y = 1450$

This system of equations lends itself to solving by the elimination/addition method. If we multiply the bottom equation by –10, the *y*-terms will cancel. This gives us:

New Equations

$x + y = 20000 \rightarrow x + y = 20000$

$-10(.05x + .10y = 1450) \rightarrow -.5x + -1y = -14500$

$.5x = 5500$

$x = \$11000$

Now we can plug 11,000 into the top equation and solve for *y.*

$11000 + y = 20000$

$11000 - 11000 + y = 20000 - 11000$

$y = \$9000$

So the 5% account had $11,000 in it and the 10% account had $9000 in it.

Example 3

Let's look at one more example where systems of equations can be used. Let's say you are flying in a plane to a town that is 1500 miles away. On the way to the town, your plane has a tailwind and it only takes you 3 hours to get there. On the way home, your plane has to deal with the same wind speed, except this time it is a headwind, so it takes you 5 hours to get home. You would like to know how fast the plane would be traveling in air with no wind (assuming that the speed in still air was the same both directions), and how fast the wind was blowing.

We first need to recognize that we have two trips, so we can make an equation for both trips. Let *p* = the speed of the plane in still air and *w* = the speed of the wind. When you have a tail wind, the wind speed pushes you, and the actual speed you travel in still air is the sum of the plane and the wind or *p + w*. The speed with the tailwind $= \frac{\text{distance}}{\text{time}} = \frac{1500}{3} = 500mph$.

When you have a headwind your speed is slowed down by the speed of the headwind and the actual speed is the difference between the speed you travel in still air and the wind speed, or *p – w*. The speed with the headwind $= \frac{\text{distance}}{\text{time}} = \frac{1500}{5} = 300mph$. This gives us two equations, one for the tailwind and one for the headwind:

Tailwind: $p + w = 500$

Head wind: $\underline{p - w = 300}$

Notice that we do not have to multiply by anything because if we add the two equations the *w's* will disappear.

$p + w = 500$

$\underline{p - w = 300}$

$2p \quad = 800$

$p \quad = 400$

Now we can plug in 400 for *p* in the first equation and solve for *w*.

$400 + w = 500$

$w = 100$

The plane would be traveling 400 mph in still air and the wind is 100mph. What a fast wind!