Square Roots Operations

Earlier, we learned how to simplify square root like $\sqrt{12} = 2\sqrt{3}$. Today, we will learn how to multiply square roots and combine like square roots.

Combine Like Square Roots

We all know how to do this:

$$x + x = 2x$$

We call *x* a variable, meaning *x* could be any number, 1, 2, 3, 2.5, a fraction... It could also be a square root, like:

$$\sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

[**Example 1**] Simplify $3\sqrt{2} - 2\sqrt{3} - \sqrt{2} - \sqrt{3}$

[**Solution**] $3\sqrt{2} - 2\sqrt{3} - \sqrt{2} - \sqrt{3} = 3\sqrt{2} - \sqrt{2} - 2\sqrt{3} - \sqrt{3} = 2\sqrt{2} - 3\sqrt{3}$

This problem is similar to $3x - 2y - x - y = 2x - 3y$.

[**Example 2**] Simplify $\sqrt{2} + \sqrt{3}$

[**Solution**] Just like when we try to simplify $x + y$, we cannot simplify $\sqrt{2} + \sqrt{3}$. Leave it as it is.

[**Example 3**] Simplify $\sqrt{48} + \sqrt{75}$

[**Solution**] This example is different from Example 2. We actually CAN simplify this expression:

$$\begin{aligned} &\sqrt{48} + \sqrt{75} \\ &= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} + \sqrt{3 \cdot 5 \cdot 5} \\ &= 2 \cdot 2\sqrt{3} + 5\sqrt{3} \\ &= 4\sqrt{3} + 5\sqrt{3} \\ &= 9\sqrt{3} \end{aligned}$$

The lesson learned is: Always try to simplify each radical first.

[**Example 4**] Simplify $\sqrt{2}(\sqrt{18}-\sqrt{10})$

[**Solution**]

$$
\begin{aligned}
&\sqrt{2}(\sqrt{18}-\sqrt{10})\\
&=\sqrt{2}\sqrt{18}-\sqrt{2}\sqrt{10}\\
&=\sqrt{36}-\sqrt{20}\\
&=6-\sqrt{2\cdot 2\cdot 5}\\
&=6-2\sqrt{5}
\end{aligned}
$$

[**Example 5**] Simplify $(\sqrt{6}-\sqrt{5})(\sqrt{2}-\sqrt{15})$

[**Solution**] We need to use FOIL first, then simplify each square root, and finally try to combine them.

$$
\begin{aligned}
&(\sqrt{6}-\sqrt{5})(\sqrt{2}-\sqrt{15})\\
&=\sqrt{6}\sqrt{2}+\sqrt{6}(-\sqrt{15})+(-\sqrt{5})\sqrt{2}+(-\sqrt{5})(-\sqrt{15})\\
&=\sqrt{12}-\sqrt{90}-\sqrt{10}+\sqrt{75}\\
&=\sqrt{2\cdot 2\cdot 3}-\sqrt{2\cdot 3\cdot 3\cdot 5}-\sqrt{2\cdot 5}+\sqrt{3\cdot 5\cdot 5}\\
&=2\sqrt{3}-3\sqrt{10}-\sqrt{10}+5\sqrt{3}\\
&=7\sqrt{3}-4\sqrt{10}
\end{aligned}
$$

[**Example 6**] Simplify $(\sqrt{3}-\sqrt{2})^2$

[**Solution**] Don't do this: $(\sqrt{3}-\sqrt{2})^2=3-2=1$. Don't! Don't! Don't!

The first step is: $(\sqrt{3}-\sqrt{2})^2=(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})$.

Next, we need to either FOIL, or use the area model, or use the perfect square formula. I will show all three methods.

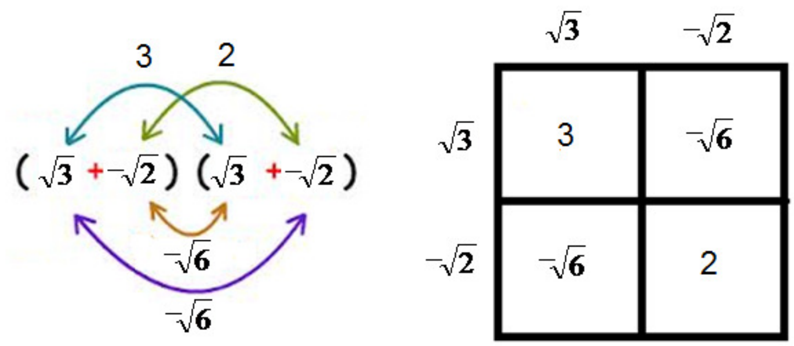

Figure 1: FOIL and area model to multiply radicals

Both methods give:

$$\begin{aligned}&(\sqrt{3}-\sqrt{2})^2\\&=(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})\\&=3-\sqrt{6}-\sqrt{6}+2\\&=5-2\sqrt{6}\end{aligned}$$

We could also use the perfect square formula:

$$\begin{aligned}(a-b)^2&=a^2-2ab+b^2\\(\sqrt{3}-\sqrt{2})^2&=(\sqrt{3})^2-2\sqrt{3}\sqrt{2}+(\sqrt{2})^2\\&=3-2\sqrt{6}+2\\&=5-2\sqrt{6}\end{aligned}$$

We used the fact that square root and square cancel out each other: $(\sqrt{x})^2=x$, if *x* is positive.

At this point, I recommend using the perfect square formula.

[**Example 7**] Simplify $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$

[**Solution**] We will use the Difference of Squares formula $(a+b)(a-b)=a^2-b^2$.

$$\begin{aligned}&(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})\\&=(\sqrt{3})^2-(\sqrt{2})^2\\&=3-2\\&=1\end{aligned}$$

When the Difference of Squares formula applies, squares in the formula will cancel out square roots.