

Special Product of Polynomials

Two types of polynomial products are used so often later that they deserve an extra lesson.

Product of Perfect Squares

[Example 1] Expand $(x + 3)^2$.

[Solution] A very common mistake is: $(x + 3)^2 = x^2 + 3^2 = x^2 + 9$.

This is wrong because $(1 + 2)^2 = 3^2 = 9$, but $1^2 + 2^2 = 1 + 4 = 5$. So $(1 + 2)^2 \neq 1^2 + 2^2$.

We need to use the definition of square first and get $(x + 3)^2 = (x + 3)(x + 3)$. Next, we use FOIL or the area model to do multiplication:

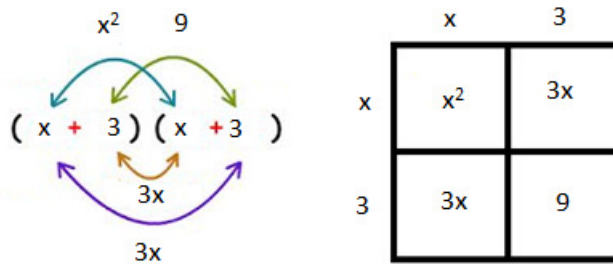


Figure 1: expand $(x+3)^2$ by FOIL and area model

Solution: $(x + 3)^2 = x^2 + 6x + 9$

Let's look for a pattern:

$$(x + 1)^2 = x^2 + 2x + 1$$

$$(x + 2)^2 = x^2 + 4x + 4$$

$$(x + 3)^2 = x^2 + 6x + 9$$

$$(x + 4)^2 = x^2 + 8x + 16$$

$$(x + 5)^2 = x^2 + 10x + 25$$

...

The pattern is:

$$\text{Formula 1: } (a + b)^2 = a^2 + 2ab + b^2$$

[Example 2] Expand $(x + 10)^2$.

[Solution] Let's use the formula:

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 \\(x + 10)^2 &= x^2 + 2 \cdot x \cdot 10 + 10^2 \\ &= x^2 + 20x + 100\end{aligned}$$

There is a related formula:

$$\text{Formula 2: } (a - b)^2 = a^2 - 2ab + b^2$$

You can easily prove this formula by FOIL.

Note that a and b in the formula could be any monomial.

[Example 3] Expand $(2x - 3y)^2$.

[Solution] Let's use the second formula:

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2 \\(2x - 3y)^2 &= (2x)^2 - 2(2x)(3y) + (3y)^2 \\ &= 4x^2 - 12xy + 9y^2\end{aligned}$$

Difference of Squares Formula

[Example 4] Expand $(x + 3)(x - 3)$.

[Solution] We use FOIL or the area model to do this multiplication:

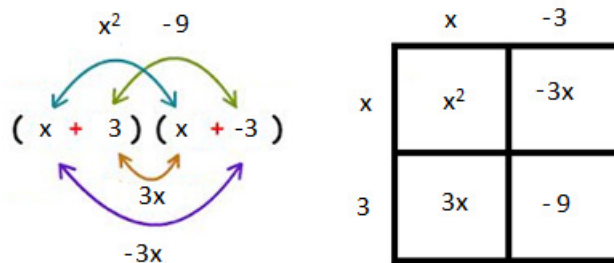


Figure 2: graphs of FOIL and area model for $(x+3)(x-3)$

$$\text{Solution: } (x + 3)(x - 3) = x^2 - 9$$

This product is special in that the middle term cancelled out. The formula is:

$$\text{Formula 3: } (a + b)(a - b) = a^2 - b^2$$

This is one of the most used formulas in mathematics.

[Example 5] Expand $(2x - 4y)(2x + 4y)$.

[Solution] Let's use Formula 3:

$$\begin{aligned} & (2x - 4y)(2x + 4y) \\ &= (2x)^2 - (4y)^2 \\ &= 4x^2 - 16y^2 \end{aligned}$$

Note that $(2x - 4y)(2x + 4y) = (2x + 4y)(2x - 4y)$, as $2 \cdot 3 = 3 \cdot 2$.

In this lesson, we learned 3 formulas:

$$\text{Formula 1: } (a + b)^2 = a^2 + 2ab + b^2$$

$$\text{Formula 2: } (a - b)^2 = a^2 - 2ab + b^2$$

$$\text{Formula 3: } (a + b)(a - b) = a^2 - b^2$$

You could use FOIL each time to do the multiplication without using these formulas. However, your life would be much easier if you can memorize and use these formulas.