Solving Systems by Elimination/Addition Method

Let's try to solve this system equation: $\begin{cases} x + y = 10 \\ x - y = 6 \end{cases}$.

We could solve this by graphing, but it takes too much work.

We could change the first equation to $x = 10 - y$, and then use substitution, but there is an easier way.

Let's add up these two equations. We have:

$$x + y + x - y = 10 + 6$$
$$2x = 16$$
$$x = 8$$

Next, plug $x = 8$ into $x + y = 10$, and we will get $y = 2$. The solution of the system equation is (8, 2).

This new method is called the elimination method, or addition method. Once we add up both equations, either *x* terms or *y* terms are eliminated, and we can solve for the variable left.

The elimination method will be the preferred method to use to solve a system of equations, if both equations are in standard form, *Ax + By = C*. Both equations need to be in standard form to use this method. This method also has 5 steps.

1) Multiply one or both equations (if necessary to eliminate a variable) by numbers that will make one of the variables disappear when you add the equations together. This will make two new equations.
2) Add the two new equations.
3) Solve for remaining variable.
4) Substitute the answer for the variable found in step 3 in one of the equations and solve to find the other variable.
5) Write the answer as an ordered pair.

Example 1

Let's look at an example: Solve the system equation $\begin{cases} 5x - 3y = 7 \\ 2x + 4y = 8 \end{cases}$

For the first step, we need to notice that if we added the two equations straight down, neither of the variables would disappear, so we need to multiply one or both of the variables by numbers that will make it so that when we add the equations one of the variables will be eliminated. You can choose either *x* or *y* to eliminate. For this example, let's eliminate the *y*'s. To eliminate the *y* variables, we need to look at the numbers in front of the *y* values (called the **coefficient** of *y*), -3 and 4. What is the smallest number that 3 and 4 divide into? 12, so we will need to multiply the equations by values that will make it so one equation has a *12y* and the other equation has a *–12y*. We can accomplish this by multiplying the first equation by 4 and the second equation by 3 as seen here:

New Equations

$4(5x - 3y = 7) \longrightarrow 20x - 12y = 28$

$3(2x + 4y = 8) \longrightarrow 6x + 12y = 24$

Now that we have our new equations, we need to add them together to get:

$20x - \cancel{12y} = 28$

$\underline{6x + \cancel{12y} = 24}$

$26x = 52$

Notice that the *y*-terms cancel and we are left with an equation that is easy to solve. Just divide both sides by 26 to get:

$x = 2$

We have now completed steps 1 – 3, so we need to substitute this answer of $x = 2$ into either of the two equations and solve for *y*. Let's plug it into the first original equation and solve.

$5(2) - 3y = 7$

$10 - 3y = 7$

$10 - 10 - 3y = 7 - 10$

$-3y = -3$

$y = 1$ Our answer is (2, 1).

Example 2

Let's look at one more example: Solve the system equation $\begin{cases} 3x+4y=7 \\ 6x+3y=24 \end{cases}$

For the first step, we need to notice that if we added the two equations straight down, neither of the variables would disappear, so we need to multiply one or both of the variables by numbers that will eliminate one of the variables when we add the equations. You can choose either *x* or *y* to eliminate. For this example, let's eliminate the *x*'s. To eliminate the *x* variables, we need to look at the numbers in front of the *x* values (called the **coefficient** of *x*), 3 and 6. What is the smallest number that 3 and 6 divide into? 6, so we will need to multiply the equations by values that will make it so one equation has a *6x* and the other equation has a *–6x*. We can accomplish this by multiplying the first equation by -2 and the second equation already has a *6x*, so we can leave it alone as seen here:

New Equations

$-2(3x + 4y = 7) \longrightarrow -6x + -8y = -14$

$6x + 3y = 24 \longrightarrow 6x + 3y = 24$

Now that we have our new equations, we need to add them together to get:

$-\cancel{6}x + -8y = -14$

$\underline{\cancel{6}x + 3y = 24}$

$-5y = 10$ Notice that the *x*-terms cancel and we are left with an equation that is easy to solve. Just divide both sides by -5 to get:

$y = -2$

We have now completed steps 1 – 3, so we need to substitute this answer of $y = -2$ into either of the two equations and solve for *x*. Let's plug it into the first original equation and solve.

$3x + 4(-2) = 7$

$3x + -8 = 7$

$3x + -8 + 8 = 7 + 8$

$3x = 15$

$x = 5$ Our answer is (5, -2).

Example 3

Solve the system $\begin{cases} x + y = 1 \\ -x - y = 1 \end{cases}$

If we add up both equations, we have $0 = 2$, which is false. This implies these two lines are parallel and thus do not intersect. There is no solution for this system.

Example 4

Solve the system $\begin{cases} x + y = 1 \\ -x - y = -1 \end{cases}$

If we add up both equations, we have $0 = 0$, which is true. This implies these two equations represent the same line. Indeed, if you multiply each term in the first equation by -1, we would get the second equation.

In this case, these two lines overlap each other. We say this system has infinitely many solutions, because they "intersect" at infinitely many points.