Solving Systems by Substitution

The second method is the substitution method. The word substitution means to replace something with something else. This can easily be done if you have solved for one of the variables. If you already have a variable by itself, then this is a good method to choose because the first step is done. There are 5 steps to this method:

1) Solve for one variable in one equation.
2) Substitute the variable into the **other** equation.
3) Solve for the first variable.
4) Substitute the answer for the first variable in either equation to find the other variable.
5) Write the answer as an ordered pair.

Let's look at an example where substitution works nicely:

Example 1

$$3x - 4y = 1$$
$$x = 2y - 1$$

Notice that *x* is already solved for in the second equation, so the first step is already done. We can replace the *x* in the first equation with the *2y-1* that the *x* equals in the second equation to get:

$3(2y - 1) - 4y = 1$ Now simplify by distributing and combining like terms.

$6y - 3 - 4y = 1$

$2y - 3 = 1$ Now add 3 to move the –3.

$2y - 3 + 3 = 1 + 3$

$2y = 4$ Now divide both sides by 2 to get:

$y = 2$

We have now finished step 3 since we have solved for the first variable. Step 4 has us substitute this answer of $y = 2$ into either equation to solve for the other variable. Let's substitute into the second equation of $x = 2y - 1$.

$x = 2(2) - 1 = 4 - 1 = 3$.

Now that we have the second variable, $x = 3$, we just need to write our answer as an ordered pair $(x, y) = (3, 2)$. Figure 1 shows the solution of Example 1.

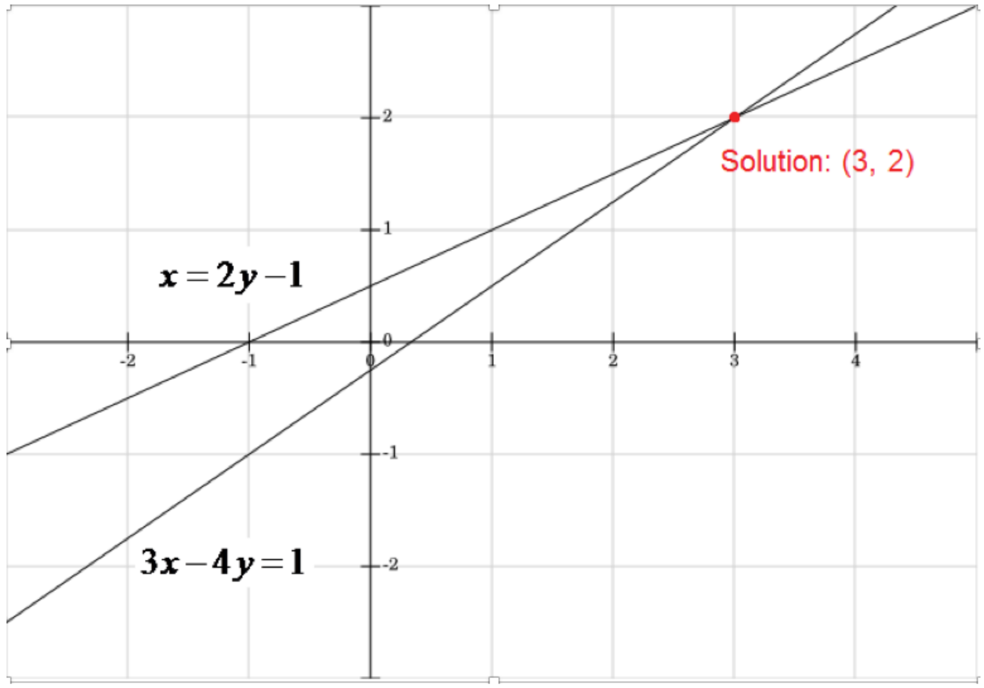

Figure 1: Solution of Example 1

Example 2

$$y = \frac{1}{4}x + 3$$

$$x - 4y = 5$$

Notice that the first equation already has a variable solved for, so step one is done and we can substitute $\frac{1}{4}x + 3$ into the second equation for the y-variable to get:

$$x - 4\left(\frac{1}{4}x + 3\right) = 5$$

Now, let's change our subtraction to adding the opposite, distribute, then combine like terms.

$$x + -4\left(\frac{1}{4}x + 3\right) = 5$$

$$x + -4\left(\frac{1}{4}x\right) + -4(3) = 5$$

$$x + -1x + -12 = 5$$

$$-12 \neq 5$$

Notice that all of the variables disappeared and the statement remaining is false. We did not do anything wrong; this means that the lines are **parallel,** which means that the system does not have any solutions. The lines will never cross. Figure 2 shows the solution of Example 2.

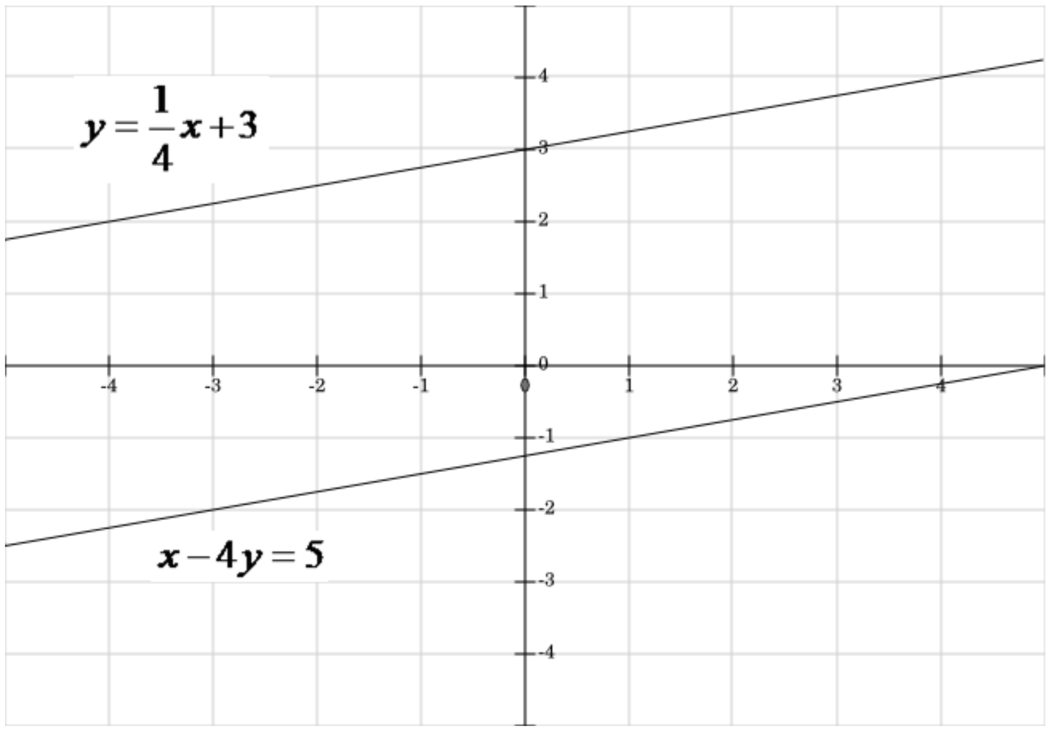

Figure 2: Solution of Example 2

Example 3

$$3y = 2x - 6$$
$$2x - 3y = 6$$

Notice that neither equation has a letter already solved for, but in the first equation it would be easy to solve for y by dividing everything by 3 to get:

$$y = \frac{2}{3}x - 2$$

Now substitute into the other equation and solve to get:

$$2x - 3\left(\frac{2}{3}x - 2\right) = 6$$
$$2x + -3\left(\frac{2}{3}x - 2\right) = 6$$
$$2x + -2x + 6 = 6$$
$$6 = 6$$

Notice that again the variables disappeared and the statement left is true. We did not do anything wrong, this means that the equations are the same equation. The equations just look different. There are an infinite number of solutions to this system. Figure 3 shows the solution of Example 3.

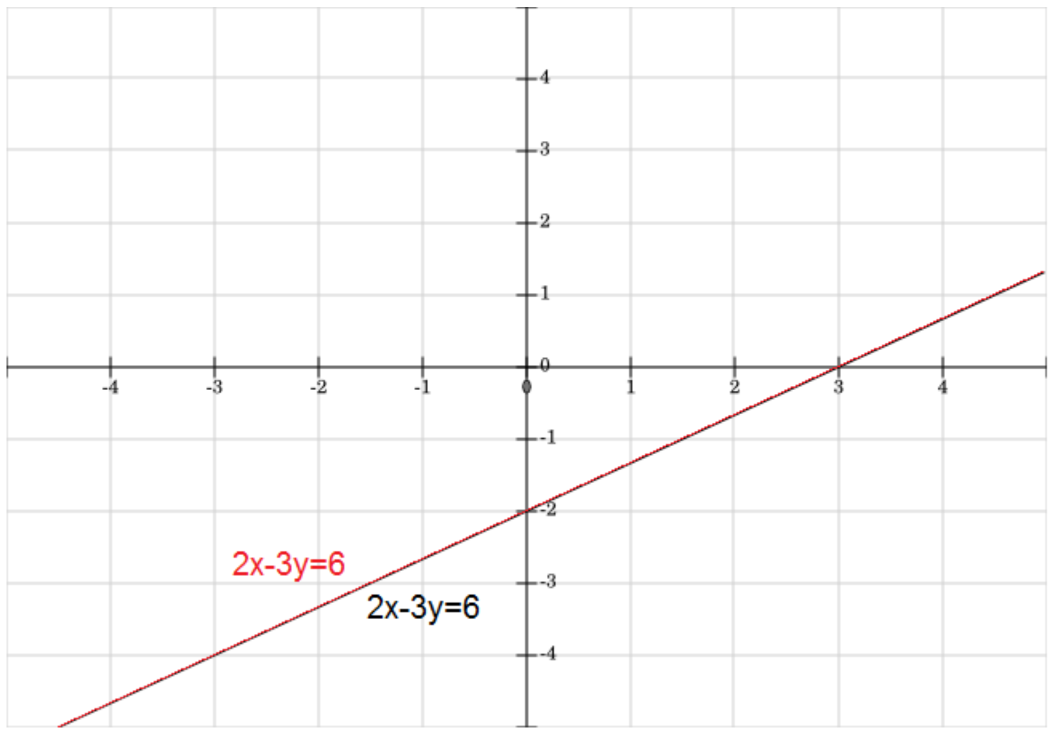

Figure 3: Solution of Example 3

When solving algebraically using the substitution method or the next method, the Elimination/Addition Method, and you have the situations that we encountered in examples 2 or 3 with the substitution method, you will have either infinite or no solutions depending on whether the remaining statement is true or false.