Solve System Equations by Graphing

Tom has $15 in his piggy bank, and he spends $1 every day. Jerry has $3 in his piggy bank, and he saves $2 every day. After how many days will they have the same amount of money?

We can easily solve this problem by guess and check. However, let's learn how to solve this problem using system equations.

We can model Tom's money by $y = -x + 15$, and model Jerry's money by $y = 2x + 3$. In both equations, *x* represents the number of days passed, and *y* represents the money in Tom's or Jerry's piggy bank.

Let's graph both lines in the same coordinate system:

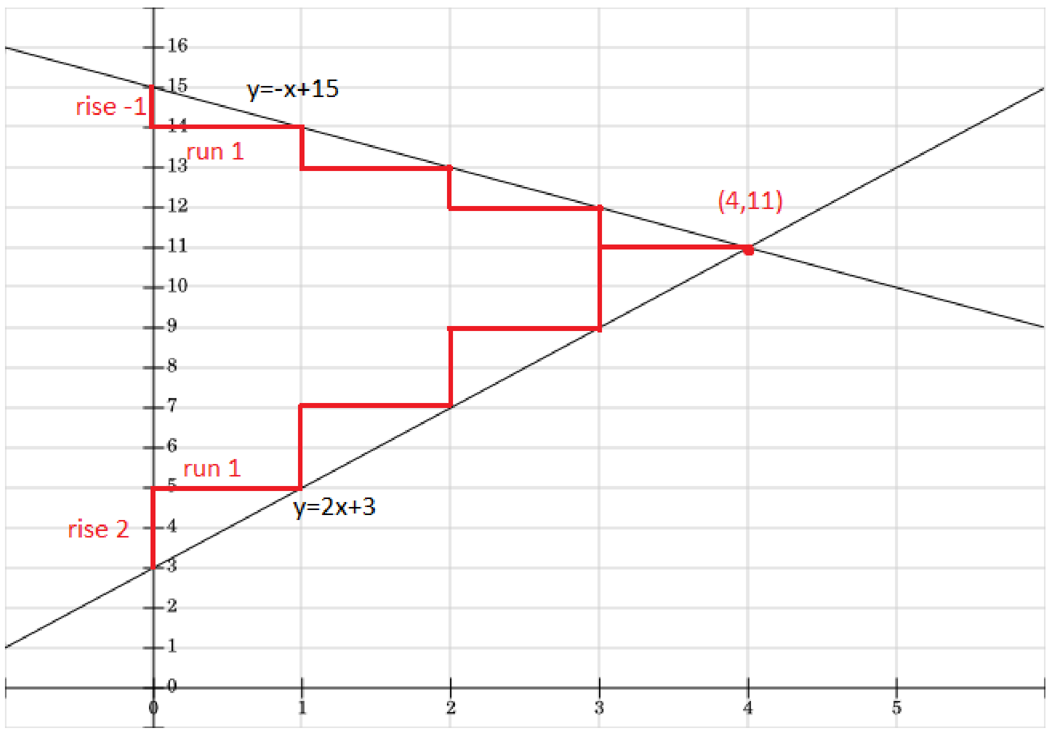

Figure 1: graphs of y=-x+15 and y=2x+3

These two lines meet at the point (4, 11), implying 4 days later, both Tom and Jerry would have $11 in their piggy bank.

This is how we solve a system equation by graphing. Note that it's wise to draw slope triangles until we find the intersection. This is more reliable than using a ruler.

[**Example 1**] Solve the system equation $\begin{cases} 2x-3y=0 \\ y=\frac{1}{3}x+3 \end{cases}$

[**Solution**] To graph the first line, we change it to slope-intercept form:

$$\begin{aligned} 2x-3y&=0 \\ 2x-3y-2x&=0-2x \\ -3y&=-2x \\ \frac{-3y}{-3}&=\frac{-2x}{-3} \\ y&=\frac{2}{3}x \end{aligned}$$

Next, we graph both lines by slope triangles:

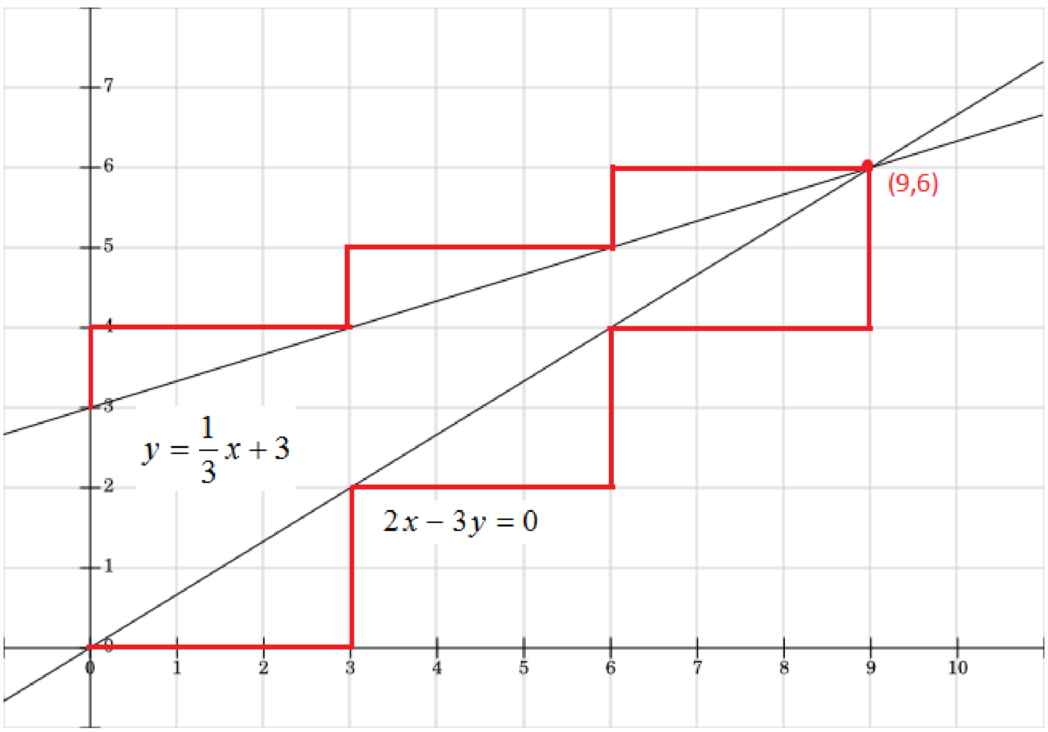

Figure 2: graphs of y=(1/3)x+3 and 2x-3y=0

Solution: By the graph, the solution for the system is (9, 6).

Again, it's critical to use slope triangles in the graph. Otherwise we could get inaccurate graphs.

The next two examples show special cases when we solve a system equation.

[**Example 2**] Solve this system equation by graphing: $\begin{cases} y = 2x + 3 \\ y = 2x - 2 \end{cases}$

[**Solution**] We graph both lines by slope triangles:

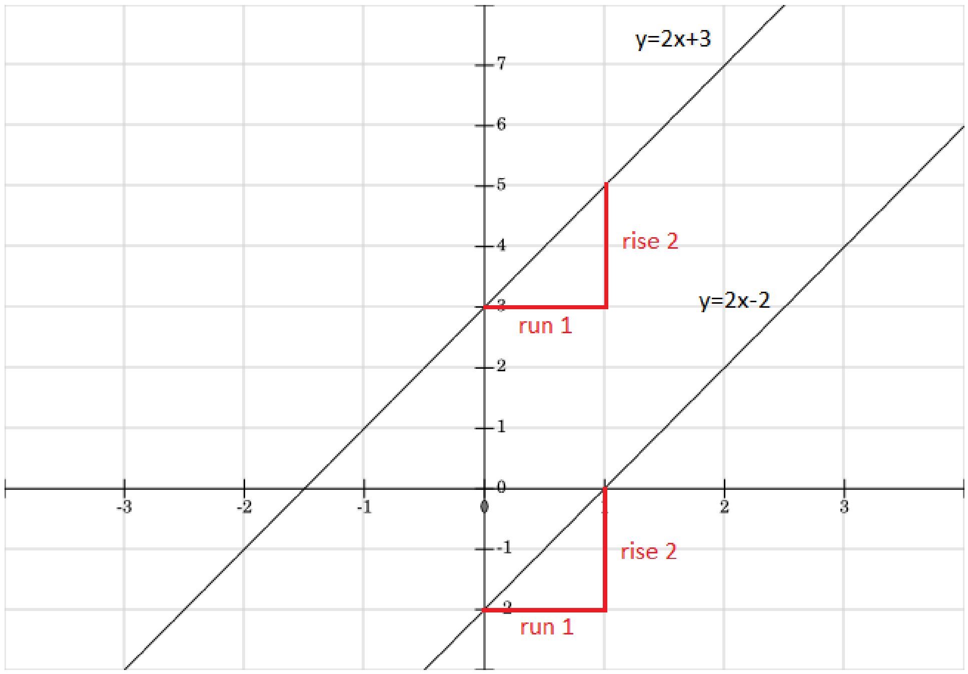

Figure 3: graphs of y=2x-2 and y=2x+3

Solution: Since these two lines have the same slope, they are parallel. This implies these two lines don't intersect, so the system equation $\begin{cases} y = 2x + 3 \\ y = 2x - 2 \end{cases}$ has no solution.

Example 3 is on the next page.

[**Example 3**] Solve the system equation $\begin{cases} y = 2x + 2 \\ 4x - 2y = -4 \end{cases}$

[**Solution**] The first line can be graphed by slope triangles as in Example 2.

We can use the intercept method to graph $4x - 2y = -4$.

Plug in *x*=0, we have -2*y*=-4 and *y*=2, so the line's *y*-intercept is (0, 2).

Plug in *y*=0, we have 4*x*=-4 and *x*=-1, so the line's *x*-intercept is (-1, 0).

Now let's see where these two lines intersect:

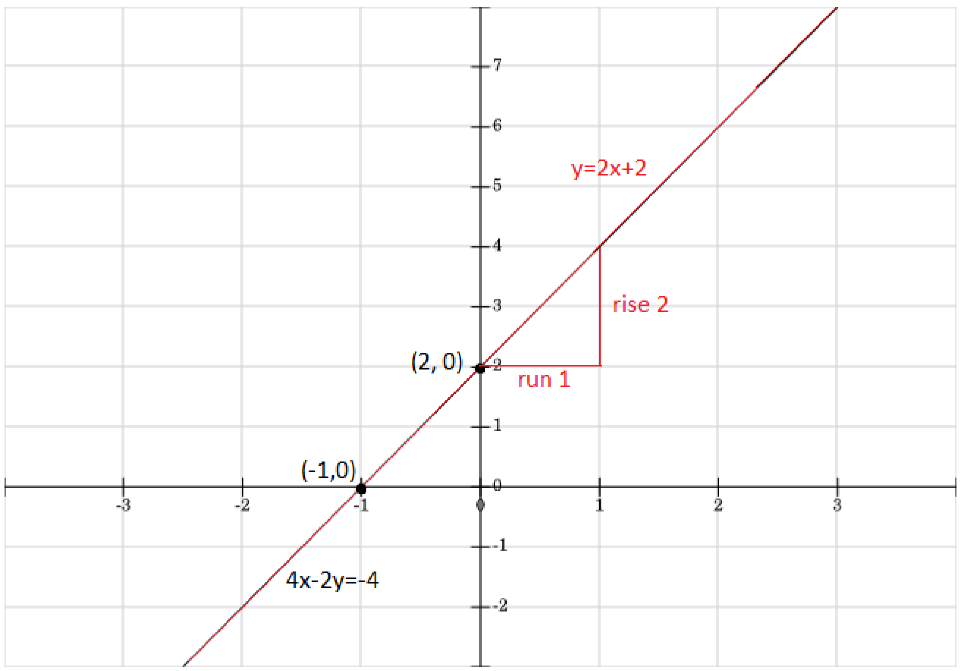

Figure 4: graphs of y=2x+2 and 4x-2y=-4

Solution: These two lines happen to be the same line! In this case, these two lines have infinitely many "intersections". We say the system $\begin{cases} y = 2x + 2 \\ 4x - 2y = -4 \end{cases}$ has infinitely many solutions.

In this section, we learned how to solve a system equation by graphing. There are two special cases.

- A system equation has no solution when those two lines are parallel.
- A system equation has infinitely many solutions when those two lines happen to be the same line.