Solving Quadratic Equations With the Square Root Property

In this unit, we will learn how to solve quadratic equations.

So far, you know how to solve linear equations, such as $2(x-2)+10=-20$. In math and science, we have to solve more complicated equations. Quadratic equations involve x^2.

In this lesson, we learn the easiest type of quadratic equations, where we can get rid of the square by using square root. Let's review this property from the Square Root unit:

$\sqrt{x^2} = x$, where *x* is positive

So, when we solve this equation:

$$x^2 = 4$$

we can square root both sides, and have:

$$\sqrt{x^2} = \sqrt{4}$$
$$x = 2$$

It looks correct, because we can plug $x=2$ into $x^2 = 4$, and it works.

Not quite... :)

Think about $x=-2$. This solution also works for $x^2=4$, as $(-2)^2=4$.

Here is the lesson: When we square root both sides of an equation, we must add in the symbol $\pm$, meaning "positive or negative":

$$x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{4}$$
$$x = \pm 2$$

There are two solutions for $x^2=4$, they are $x=2$ and $x=-2$, also written as $x=\pm 2$.

Let's look at a few examples.

[**Example 1**] Solve $5x^2 = 20$ for x

[**Solution**] First divide both sides by 5 to get the x^2 by itself:

$$\frac{5x^2}{5} = \frac{20}{5}$$
$$x^2 = 4$$

Now take the square root of both sides using a $\pm$ in front of the number.

$$\sqrt{x^2} = \pm\sqrt{4}$$
$$x = \pm 2$$
$$\text{or}$$
$$x = -2 \text{ and } 2$$

[**Example 2**] Solve $3x^2 - 24 = 0$ for x

[**Solution**] First add 24 to both sides, then divide by 3 to get the x^2 by itself.

$$3x^2 - 24 + 24 = 0 + 24$$
$$3x^2 = 24$$
$$\frac{3x^2}{3} = \frac{24}{3}$$
$$x^2 = 8$$

Now, we need to take the square root of both sides and use a $\pm$ on the side with the number.

$$\sqrt{x^2} = \pm\sqrt{8}$$
$$x = \pm\sqrt{8} = \pm\sqrt{4 \bullet 2} = \pm\sqrt{4}\sqrt{2} = \pm 2\sqrt{2}$$
$$x = \pm 2\sqrt{2}$$
$$\text{or}$$
$$x = -2\sqrt{2} \text{ and } 2\sqrt{2}$$

Notice that on this one we had to simplify the square root.

Let's look at a slightly different problem that uses the same technique.

[**Example 3**] Solve $(x+4)^2 = 25$ for x

[**Solution**] Notice that more than just *x* is squared, yet what is squared is by itself, so we can go ahead and take the square root of both sides to get:

$$\sqrt{(x+4)^2} = \pm\sqrt{25}$$
$$x + 4 = \pm 5$$

Now notice that to solve for *x* we have to add a negative 4 to both sides. When we add the –4 to the right side, we will insert it in front of the $\pm$, as seen here:

$$x+4+-4=-4\pm 5$$
$$x=-4\pm 5, \text{ so}$$
$$x=-4+5=1$$
and
$$x=-4-5=-9$$

Notice that this time we had the two answers of $x = 1$ and -9.

[**Example 4**] Solve $x^2=-4$ for x

[**Solution**] We have the x^2 by itself, so we just need to square root both sides. But wait! We cannot square root a negative number at this time, so there are “No Real Solutions.”

Application of Solving Quadratic Equation

Now let’s look at a situation where these equations and square roots would be helpful. Recently I built an overhang on a barn at my farm. I figured out how much and what kind of wood and metal I needed, called up the local lumber store and ordered it. Later I was out measuring everything (I had just estimated when figuring out my material. I remembered most of the measurements.) when I realized that the drop of the overhang was larger than I had planned. Suddenly I was not sure that the 16 foot boards I had ordered would be long enough, so I had to refer to my geometry and square root skills to make sure my boards and metal would work. The overhang looks like this with an imaginary triangle that I can draw on the top as seen labeled with measurements in the picture.

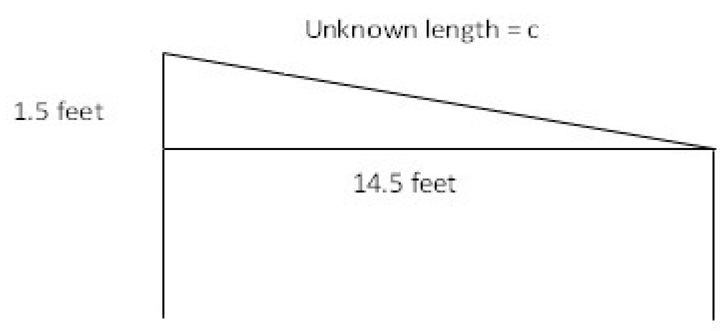

Figure 1: graph of Pythagorean Theorem application

There is a nice theorem about right triangles called the Pythagorean Theorem that states that the sum of the square of the legs of a right triangle is equal to the square of the hypotenuse, or $a^2 + b^2 = c^2$, using the variables as seen in the right triangle below.

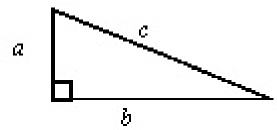

Figure 2: Pythagorean Theorem

If we plug the measurements from my barn in, we can use the skills from this section to determine if the boards are long enough and if I might even have extra length that will hang over beyond the posts a bit. Using Pythagorean Theorem, we have:

$$
\begin{aligned}
&(1.5)^2 + (14.5)^2 = c^2 \\
&2.25 + 210.25 = c^2 \\
&212.5 = c^2 \\
&\sqrt{212.5} = \sqrt{c^2} \\
&14.6 \text{ feet } \approx \text{c}
\end{aligned}
$$

The boards were long enough! Notice that in this situation we did not use the $\pm$, because it did not make sense to have a negative length on a board.