Solving Quadratic Equations by Factoring

In the last lesson, we learned how to solve a quadratic equation by using the square root property. Again: Don't forget to add the $\pm$ symbol when you square root both sides of an equation!

Unfortunately, the square root property doesn't work in all situations. Look at this problem:

Solve $x^2 + x = 6$ for x

If we still try to use the square root property, we will have:

$$\sqrt{x^2 + x} = \pm\sqrt{6}$$

and we are stuck! The square root cannot cancel the square because of the plus sign!

We need to learn a new way to solve quadratic equations: by factoring.

First, we need to learn the **Zero Product Property**:

If *ab = 0*, then either *a = 0* or *b = 0*.

Basically if you multiply two numbers, the only way to get zero is if one of the numbers is zero.

Now we are ready to solve $x^2 + x = 6$ for *x*. There are 4 basic steps to solving equations by factoring:

1. Simplify both sides of the equation.
2. Get zero on one side of the equation.
3. Factor the polynomial.
4. Break the problem up by setting each factor equal to zero and solve. This step uses Zero Product Property.

[**Example 1**] Solve $x^2 + 3x - 4 = 0$ for x

[**Solution**] Notice that both sides of the equation are simplified and that we already have zero on one side, so all we need to do is factor (Step 3). To factor this trinomial, we ask: What multiplies to get –4 and adds up to +3? The answer is 4 and –1, so this factors into:

$$(x+4)(x-1) = 0$$

The product of two numbers, $(x+4)$ and $(x-1)$, is 0. By Zero Product Property, one of them has to be zero. So we have:

$$x+4=0 \qquad\qquad x-1=0$$
$$x+4-4=0-4 \quad \textbf{or} \quad x-1+1=0+1$$
$$x=-4 \qquad\qquad x=1$$

Notice that we have two solutions, –4 and 1. Let's check.

Plug $x=-4$ and $x=1$ into $x^2+3x-4=0$, we have:

$$\begin{aligned} x^2+3x-4&=0 \\ (-4)^2+3(-4)-4&=0 \\ 16-12-4&=0 \\ 4-4&=0 \\ 0&=0 \end{aligned} \quad \text{and} \quad \begin{aligned} x^2+3x-4&=0 \\ 1^2+3\cdot 1-4&=0 \\ 1+3-4&=0 \\ 4-4&=0 \\ 0&=0 \end{aligned}$$

Both solutions checked! The solutions of $x^2+3x-4=0$ are $x=-4$ and $x=1$.

It's not true that we always get two solutions for a quadratic equation. See Example 2.

[**Example 2**] Solve $4x(x-3)=-9$ for x

[**Solution**] The first step is to simply both sides:

$$4x^2-12x=-9$$

Next, since this equation is quadratic, we need 0 on one side:

$$4x^2-12x+9=0$$

Next step is to factor the left side. If you notice that $4x^2=(2x)^2$ and $9=3^2$, you would try to use the perfect square formula:

$$a^2 \;-\; 2\cdot\; a \quad \cdot b+b^2=(a-b)^2$$
$$4x^2-12x+9=(2x)^2-2\cdot(2x)\cdot 3+3^2=(2x-3)^2$$

So we can factor the left side of $4x^2-12x+9=0$ and have:

$$(2x-3)^2=0$$

Only 0's square is 0, so we have:

$$\begin{aligned} 2x-3&=0 \\ 2x-3+3&=0+3 \\ 2x&=3 \\ \frac{2x}{2}&=\frac{3}{2} \\ x&=\frac{3}{2} \end{aligned}$$

The only solution of $4x(x-3)=-9$ is $x=\frac{3}{2}$.

[**Example 3**] Solve $2x(x-5)=3x$ for x

[**Solution**]

We first need to simplify to get:

$$2x^2-10x=3x$$

Now we need to get zero on one side, so we will subtract *3x* from both sides of the equation.

$$2x^2-10x-3x=3x-3x$$
$$2x^2-13x=0$$

Now we need to factor. It's easy to forget that the first step of factoring is to factor out common factors. Notice that both terms have an x, so we can factor it out to get:

$$x(2x-13)=0$$

Now set each factor equal to zero and solve.

$$x=0 \quad \text{or} \quad \begin{aligned} 2x-13&=0 \\ 2x&=13 \\ x&=\frac{13}{2} \end{aligned}$$

The solutions for $2x(x-5)=3x$ are $x=0$ and $x=\frac{13}{2}$.

[**Example 4**] Solve $x^2-25=0$ for x

[**Solution**] We could solve this by factoring:

$$(x+5)(x-5)=0$$
$$x+5=0 \quad \text{or} \quad x-5=0$$
$$x=-5 \quad \text{or} \quad x=5$$

Or, we can use the square root property from an earlier lesson:

$$x^2-25=0$$
$$x^2=25$$
$$x=\pm\sqrt{25}=\pm 5$$

Just want to show more than one way to solve a quadratic equation.

Next, I'd like to point out important differences in solving several types of equations:

- When we solve a linear equation, like $2x-4=0$, we need to move all x terms to one side of the equal sign, and move all number to the other side of the equal sign. So we change $2x-4=0$ to $2x=4$. Then we can divide both sides by 2 and get the solution $x=2$.
- When we solve a quadratic equation with only x^2 terms, like $x^2-4=0$, we need to move all x^2 terms to one side of the equal sign, and move all numbers to the other side of the equal sign. We will get $x^2=4$ in this example. Then, we use the square root property to get $x=\pm\sqrt{4}=\pm 2$.
- When we solve a quadratic equation with both x^2 terms and x terms, like $x^2-3x=4$, we need to move all terms to one side of the equal sign, and make the other side 0: $x^2-3x-4=0$. Then we can try to factor and solve the equation: $(x-4)(x+1)=0$, $x-4=0$ or $x+1=0$, $x=4$ or $x=-1$.