Scientific Notation

The distance from the sun to the earth is approximately 149,600,000 kilometers. In scientific notation, it is approximately $1.5 \cdot 10^8$ kilometers.

The length of an electron is approximately 0.00000000000000282 meters. In scientific notation, it is $2.82 \cdot 10^{-15}$ meters.

We need scientific notation because it is easier than writing long numbers.

To fully understand scientific notation, let's learn a pattern.

If we were charged $1.20 ten times, we would pay a total of $12.00. We can write:

$$1.2 \cdot 10 = 12 \text{, or } 1.2 \cdot 10^1 = 12$$

Similarly, if we are charged $1.20 one hundred times, we would pay a total of $120. We can write:

$$1.2 \cdot 100 = 120 \text{, or } 1.2 \cdot 10^2 = 120$$

The pattern is: If a number is multiplied by 10^m, where *m* is a positive integer, we can simply move the decimal point to the right *m* times. For example:

$$1.2 \cdot 10^5 = 1\underbrace{20000}_{\substack{\text{move decimal} \\ \text{point to the} \\ \text{right 5 times}}}$$

We use this pattern to change big numbers into scientific notation:

$$120000 = 1\underbrace{20000.}_{\substack{\text{move decimal} \\ \text{point to the} \\ \text{left 5 times}}} = 1.2 \cdot 10^5$$

To qualify for scientific notation, the decimal must be in the range of [1,10). For example, neither $12 \cdot 10^4$ nor $0.12 \cdot 10^6$ qualify as scientific notation, even though they are equivalent to $1.2 \cdot 10^5$.

It's difficult to try to remember which side to move the decimal point. Instead of memorizing rules, understand it this way: When we change 120000 to 1.2, we made the number smaller. To compensate, we should multiply 1.2 with a big number like 10^5. This is why $120000 = 1.2 \cdot 10^5$ makes sense.

Earlier, we learned negative exponents. We learned:

$$10^{-1} = \frac{1}{10^1} = 0.1$$
$$10^{-2} = \frac{1}{10^2} = 0.01$$
$$10^{-3} = \frac{1}{10^3} = 0.001$$
$$\ldots$$

Again, a negative exponent and a negative number are two different concepts!

Now we have:

$$1.2 \cdot 10^{-1} = 1.2 \cdot 0.1 = 0.12$$
$$1.2 \cdot 10^{-2} = 1.2 \cdot 0.01 = 0.012$$
$$1.2 \cdot 10^{-3} = 1.2 \cdot 0.001 = 0.0012$$
$$\ldots$$

Now we can change a small number into scientific notation:

$$0.000012 = 0.\underbrace{00001}_{\substack{\text{move decimal} \\ \text{point to the} \\ \text{right 5 times}}}2 = 1.2 \cdot 10^{-5}$$

Again, instead of memorizing which way to move, think this way: If we change 0.000012 to 1.2, we made the number bigger. To compensate, we should multiply 1.2 by a small number like 10^{-5}.

[**Example 1**] Do multiplication $8.4 \cdot 10^5 \cdot 4.5 \cdot 10^3$ and write your answer in scientific notation.

[**Solution**]

$$\begin{aligned} & 8.4 \cdot 10^5 \cdot 4.5 \cdot 10^3 \\ = {} & 8.4 \cdot 4.5 \cdot 10^5 \cdot 10^3 \\ = {} & 37.8 \cdot 10^8 \end{aligned}$$

Be careful: You are not done because the result is not in scientific notation yet. We have to change 37.8 to a number in the range of [1,10). We have:

$$
\begin{aligned}
&8.4 \cdot 10^5 \cdot 4.5 \cdot 10^3 \\
&= 8.4 \cdot 4.5 \cdot 10^5 \cdot 10^3 \\
&= 37.8 \cdot 10^8 \\
&= 3.78 \cdot 10^1 \cdot 10^8 \\
&= 3.78 \cdot 10^9
\end{aligned}
$$

Note that when we changed 37.8 to $3.78 \cdot 10^1$, we should think this way: We changed 37.8 to 3.78, from a big number to a small number. To compensate, we have to multiply 3.78 by a big number, like 10^1. It would not make sense to multiply 3.78 by 10^{-1} to make it even smaller.

[**Example 2**] Do division $\frac{4.2 \cdot 10^{-9}}{8.4 \cdot 10^{-2}}$ and write your answer in scientific notation.

[**Solution**]

$$
\begin{aligned}
&\frac{4.2 \cdot 10^{-9}}{8.4 \cdot 10^{-2}} \\
&= \frac{4.2}{8.4} \cdot 10^{-9-(-2)} \\
&= 0.5 \cdot 10^{-7} \\
&= 5 \cdot 10^{-1} \cdot 10^{-7} \\
&= 5 \cdot 10^{-8}
\end{aligned}
$$

Similarly, when we changed 0.5 to $5 \cdot 10^{-1}$, we changed 0.5 to a bigger number 5. To compensate, we need to multiply 5 by a small number like 10^{-1}.