Rationalize Denominator

In mathematics, we do not want to see radicals in the denominator of a fraction, like in $\frac{1}{\sqrt{2}}$. We need to figure out a way to get rid of radicals from the denominator. Let's observe a pattern:

$$\sqrt{4}\cdot\sqrt{4}=2\cdot 2=4$$
$$\sqrt{9}\cdot\sqrt{9}=3\cdot 3=9$$
$$\sqrt{16}\cdot\sqrt{16}=4\cdot 4=16$$
$$\ldots$$
$$\sqrt{x}\cdot\sqrt{x}=x$$

With a calculator, you can verify $\sqrt{2}\cdot\sqrt{2}=2$. This should make sense because $\sqrt{2}$ is the number which, when multiplying by itself, produces 2. This is the definition of square root.

Now we can get rid of $\sqrt{2}$ in the denominator of $\frac{1}{\sqrt{2}}$ this way:

$$\frac{1}{\sqrt{2}}=\frac{1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{\sqrt{2}}{2}$$

In the denominator, we used the pattern we just observed: $\sqrt{2}\cdot\sqrt{2}=2$. This process is called "rationalize the denominator."

[**Example 1**] Rationalize denominator in $\frac{3}{\sqrt{5}}$

[**Solution**]

$$\frac{3}{\sqrt{5}}$$
$$=\frac{3\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}}$$
$$=\frac{3\sqrt{5}}{5}$$

Scroll down for another example.

[**Example 2**] Rationalize denominator in $\frac{1}{\sqrt{12}}$.

[**Solution**] It's always easier to simplify a square root when possible.

In this example, we will first do $\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} = 2\sqrt{3}$. We have:

$$\begin{aligned}
&\frac{1}{\sqrt{12}} \\
&= \frac{1}{2\sqrt{3}} \\
&= \frac{1 \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} \\
&= \frac{\sqrt{3}}{2 \cdot 3} \\
&= \frac{\sqrt{3}}{6}
\end{aligned}$$

By simplifying $\sqrt{12}$ first, we avoided multiplying $\sqrt{12}$ in the numerator and denominator.