## Application of Quadratic Equations

Now let's solve some word problems involving quadratic equations. Let's start with a number problem.

[**Example 1**] The sum of two number is 16, and their product is 63. Find these two numbers.

[**Solution**] We could solve this problem by guess and check, but we will use a quadratic equation instead. Assume one number is *x*. Since the sum of these two numbers is 16, the other number must be 16-*x*. Now we can write an equation by their product:

$$\begin{aligned} &x(16-x)=63 \\ &16x-x^2=63 \\ &-x^2+16x-63=0 \end{aligned}$$

To get rid of the leading negative symbol, we multiply each term by $-1$:

$$(-1)\cdot(-x^2)+(-1)\cdot 16x+(-1)\cdot(-63)=(-1)\cdot 0$$
$$x^2-16x+63=0$$

It's easy to solve this equation by factoring:

$$\begin{aligned} &(x-7)(x-9)=0 \\ &x-7=0 \quad \text{or} \quad x-9=0 \\ &x=7 \qquad\quad \text{or} \quad x=9 \end{aligned}$$

If $x=7$, the other number is $16-7=9$.

If $x=9$, the other number is $16-9=7$.

**Solution**: These two numbers are 7 and 9.

A classic type of quadratic equation word problem involves a rectangle's area.

[**Example 2**] A rectangle's length is 5 meters longer than twice its width. The rectangle's area is 75 square meters. Find this rectangle's dimensions.

[**Solution**] Assume the rectangle's width is $x$ meters, then its length is $2x+5$ meters. By the given area, we can write an equation and then solve it:

$$\begin{aligned} &x(2x+5)=75 \\ &2x^2+5x-75=0 \end{aligned}$$

Let's use the "ac method" to factor the left side. Identify that $a = 2, c = -75$, so $ac = -150$. We need to list possible 2-number product of $-150$ one by one until we find a pair whose sum is $5$:

$$
\begin{aligned}
&-1 \cdot 150 = -150 \quad 1 \cdot (-150) = -150 \\
&-2 \cdot 75 = -150 \quad 2 \cdot (-75) = -150 \\
&-3 \cdot 50 = -150 \quad 3 \cdot (-50) = -150 \\
&-4 \cdot \text{nothing} = -150 \\
&-5 \cdot 30 = -150 \quad 5 \cdot (-30) = -150 \\
&-6 \cdot 25 = -150 \quad 6 \cdot (-25) = -150 \\
&-7 \cdot \text{nothing} = -150 \\
&-8 \cdot \text{nothing} = -150 \\
&-9 \cdot \text{nothing} = -150 \\
&-10 \cdot 15 = -150 \quad 10 \cdot (-15) = -150
\end{aligned}
$$

We can stop here because we found the pair: -10 and 15. Next, let's factor the left side:

$$
\begin{aligned}
&2x^2 + 5x - 75 = 0 \\
&2x^2 - 10x + 15x - 75 = 0 \\
&2x(x-5) + 15(x-5) = 0 \\
&(2x+15)(x-5) = 0 \\
&2x + 15 = 0 \quad \text{or} \quad x - 5 = 0 \\
&2x = -15 \quad \text{or} \quad x = 5 \\
&x = -7.5 \quad \text{or} \quad x = 5
\end{aligned}
$$

Thinking back, maybe Quadratic Formula would have been easier. Anyway...

We toss out the solution $x = -7.5$, because it makes no sense if the width is a negative number.

**Solution**: The width is $5$ meters, and the length is $2 \cdot 5 + 5 = 15$ meters.

Things could get a little bit complicated involving a rectangle's area.

[**Example 3**] You have 1000 feet of fence, and you will build a field. One side of the field is already blocked by a mountain, so you only need to build 3 sides of the rectangular field. You want the area of the field to be exactly 123,750 ft$^2$. What are the dimensions of the field?

[**Solution**] Since the problem involves a rectangle, drawing a graph will always help.

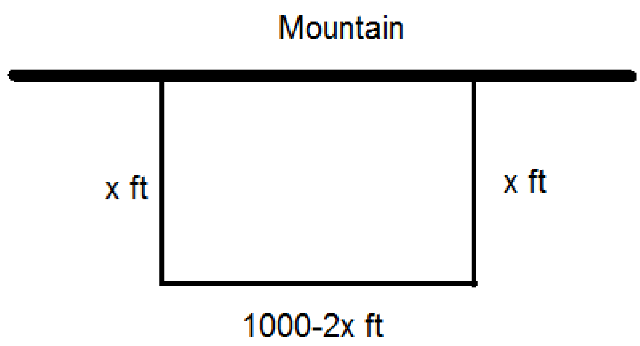


**Figure 1: a rectangular field built next to a mountain**

Assume the width of the field is *x* feet. Since you have a total of 1000 ft of fence, the length of the field must be $1000-2x$ feet. It would be difficult to see this without a graph.

Since the area of the field is 123,750 ft$^2$, we can write this equation and then solve it:

$$x(1000-2x)=123750$$
$$1000x-2x^2=123750$$
$$-2x^2+1000x-123750=0$$

Don't try to factor it, since the numbers are too big. We will use Quadratic Formula instead. Since we will use the formula anyway, there is no need to get rid of the leading negative symbol, and there is no need to divide all terms by 2 to make the equation simpler. Quadratic Formula will take care of all these issues. We identify $a=-2, b=1000, c=-123750$, and we have:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-1000\pm\sqrt{1000^2-4\cdot(-2)\cdot(-123750)}}{2\cdot(-2)}$$
$$x=\frac{-1000\pm\sqrt{10000}}{-4}$$
$$x=\frac{-1000\pm100}{-4}$$
$$x=\frac{-1000+100}{-4} \quad \text{or} \quad x=\frac{-1000-100}{-4}$$
$$x=225 \quad \text{or} \quad x=275$$

If the width is 225 ft, the length would be $1000-2\cdot225=275$ ft.

If the width is 275 ft, the length would be $1000-2\cdot275=225$ ft.

**Solution**: There are two solutions. The dimensions of the field could be 225 by 275 ft, or 275 by 225 ft.