Shifting Parabola Up/Down

A parabola is the graph of a quadratic function. A quadratic function, in its standard form, looks like

$$f(x) = ax^2 + bx + c$$

Here are some basic terms about a parabola:

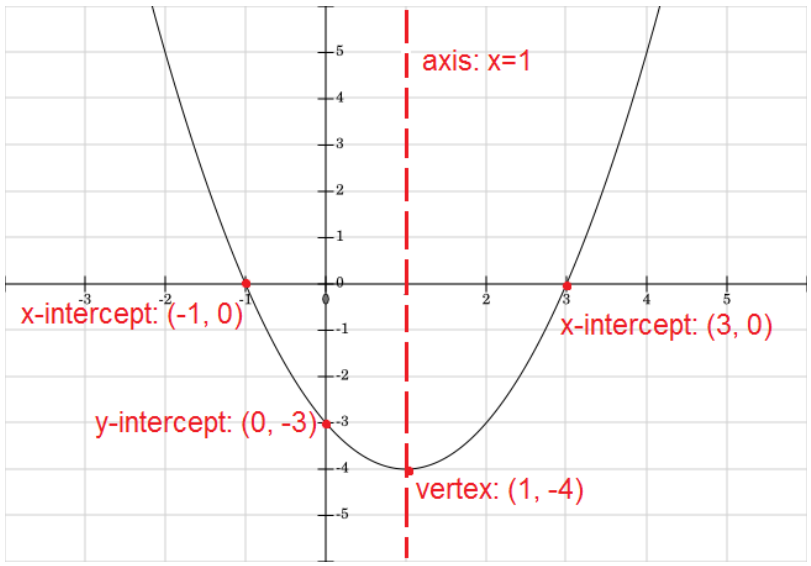

Figure 1: terms related to parabola

vertex: The highest or lowest point of a parabola is its vertex. For the parabola in the graph, the vertex is (1, -4), the lowest point. If this parabola is upside down, its vertex would be the highest point.

***y*-intercept**: A parabola crosses the *y*-axis at its *y*-intercept. A parabola has only one *y*-intercept.

***x*-intercept**: A parabola crosses the *x*-axis at its *x*-intercept(s). In the graph, the parabola has two *x*-intercepts, (-1, 0) and (3, 0). A parabola could have two, one or no *x*-intercepts, depending on the location of the parabola.

axis: The vertical line crossing a parabola's vertex is its axis, $x = 1$ in the graph. A parabola's axis is also its line of symmetry, meaning if we fold the parabola by its axis, its two sides would match.

As a starting point, let's graph the most basic quadratic function, $f(x) = x^2$.

If a function's equation is given, we can always graph it by building a table of points.

Table and graph of $f(x) = x^2$

x	y	points
-3	$y = (-3)^2 = 9$	$(-3,9)$
-2	$y = (-2)^2 = 4$	$(-2,4)$
-1	$y = (-1)^2 = 1$	$(-1,1)$
0	$y = 0^2 = 0$	$(0,0)$
1	$y = 1^2 = 1$	$(1,1)$
2	$y = 2^2 = 4$	$(2,4)$
3	$y = 3^2 = 9$	$(3,9)$

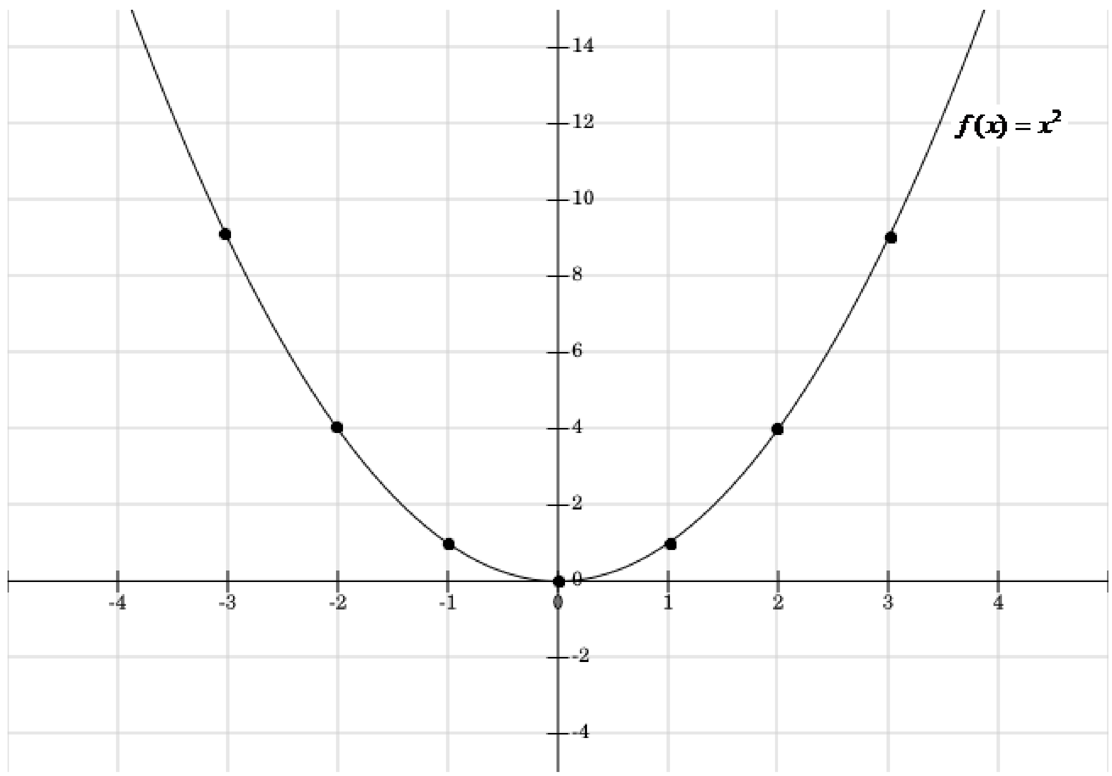

Figure 2: Graph of f(x)=x²

By the graph, the vertex of $f(x) = x^2$ is (0, 0). Its *y*-intercept is (0, 0). It has one *x*-intercept: (0, 0). Its axis is the *y*-axis, $x = 0$.

Next, let's graph the following 3 parabolas in the same graph, and find a pattern:

$f(x) = x^2$, $g(x) = x^2 + 2$, $h(x) = x^2 - 2$

Tables and Graphs of $f(x)=x^2$, $g(x)=x^2+2$, $h(x)=x^2-2$

x	$f(x)=x^2$		x	$g(x)=x^2+2$		x	$h(x)=x^2-2$
-3	$y=(-3)^2=9$		-3	$y=(-3)^2+2=11$		-3	$y=(-3)^2-2=7$
-2	$y=(-2)^2=4$		-2	$y=(-2)^2+2=6$		-2	$y=(-2)^2-2=2$
-1	$y=(-1)^2=1$		-1	$y=(-1)^2+2=3$		-1	$y=(-1)^2-2=-1$
0	$y=0^2=0$		0	$y=0^2+2=2$		0	$y=0^2-2=-2$
1	$y=1^2=1$		1	$y=1^2+2=3$		1	$y=1^2-2=-1$
2	$y=2^2=4$		2	$y=2^2+2=6$		2	$y=2^2-2=2$
3	$y=3^2=9$		3	$y=3^2+2=11$		3	$y=3^2-2=7$

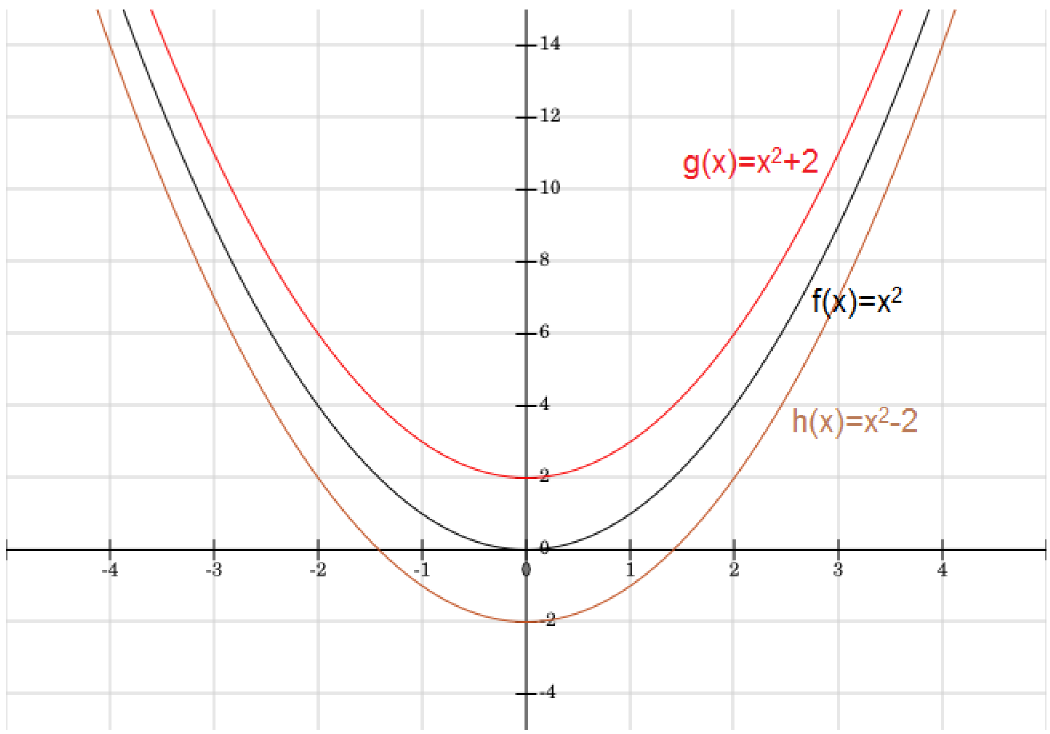

Figure 3: Graphs of f(x), g(x) and h(x)

It's easy to observe this pattern. Assume $a>0$.

If we change the function from $f(x)=x^2$ to $g(x)=x^2+a$, the parabola shifts up by a units.

If we change the function from $f(x)=x^2$ to $h(x)=x^2-a$, the parabola shifts down by a units.

Again, don't memorize these patterns. Instead, when you need them, sketch tables and graphs on scratch paper and find patterns as needed.