Thinner and Wider Parabola

Earlier, we learned that, in $f(x) = x^2 + c$, the value of c shifts the parabola up and down. Today, we will learn how a's value in $f(x) = ax^2$ will change the parabola's shape.

First, let's graph $j(x) = x^2$ and $k(x) = -x^2$ in the same coordinate system.

x	$j(x) = x^2$	points		x	$k(x) = -x^2$	points
-3	$y = (-3)^2 = 9$	(-3,9)		-3	$y = -(-3)^2 = -9$	(-3,-9)
-2	$y = (-2)^2 = 4$	(-2,4)		-2	$y = -(-2)^2 = -4$	(-2,-4)
-1	$y = (-1)^2 = 1$	(-1,1)		-1	$y = -(-1)^2 = -1$	(-1,-1)
0	$y = 0^2 = 0$	(0,0)		0	$y = -0^2 = 0$	(0,0)
1	$y = 1^2 = 1$	(1,1)		1	$y = -1^2 = -1$	(1,-1)
2	$y = 2^2 = 4$	(2,4)		2	$y = -2^2 = -4$	(2,-4)
3	$y = 3^2 = 9$	(3,9)		3	$y = -3^2 = -9$	(3,-9)

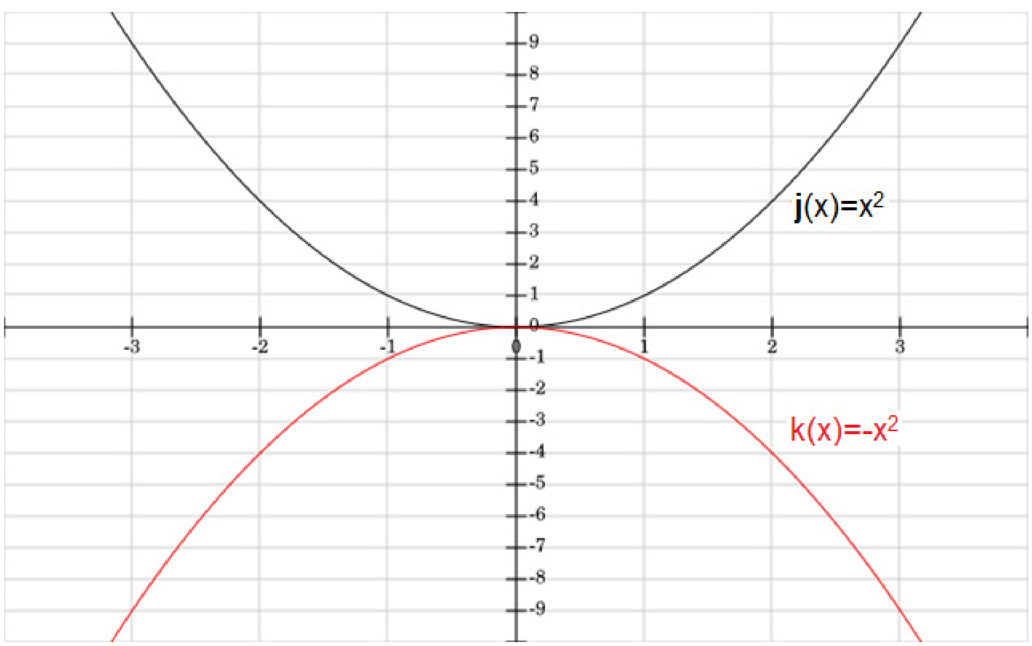

Figure 1: graphs of $j(x)=x^2$ and $k(x)=-x^2$

Here is the pattern: For $f(x) = ax^2$, when $a > 0$, the parabola faces up; when $a < 0$, the parabola faces down.

Next, let's investigate the graphs of $r(x) = 0.5x^2$, $s(x) = x^2$, $t(x) = 2x^2$.

Tables and Graphs of $r(x)=0.5x^2$, $s(x)=x^2$, $t(x)=2x^2$

x	$r(x)=0.5x^2$		x	$s(x)=x^2$		x	$t(x)=2x^2$
-3	$y=0.5(-3)^2=4.5$		-3	$y=(-3)^2=9$		-3	$y=2(-3)^2=18$
-2	$y=0.5(-2)^2=2$		-2	$y=(-2)^2=4$		-2	$y=2(-2)^2=8$
-1	$y=0.5(-1)^2=0.5$		-1	$y=(-1)^2=1$		-1	$y=2(-1)^2=2$
0	$y=0.5\cdot 0^2=0$		0	$y=0^2=0$		0	$y=2\cdot 0^2=0$
1	$y=0.5\cdot 1^2=0.5$		1	$y=1^2=1$		1	$y=2\cdot 1^2=2$
2	$y=0.5\cdot 2^2=2$		2	$y=2^2=4$		2	$y=2\cdot 2^2=8$
3	$y=0.5\cdot 3^2=4.5$		3	$y=3^2=9$		3	$y=2\cdot 3^2=18$

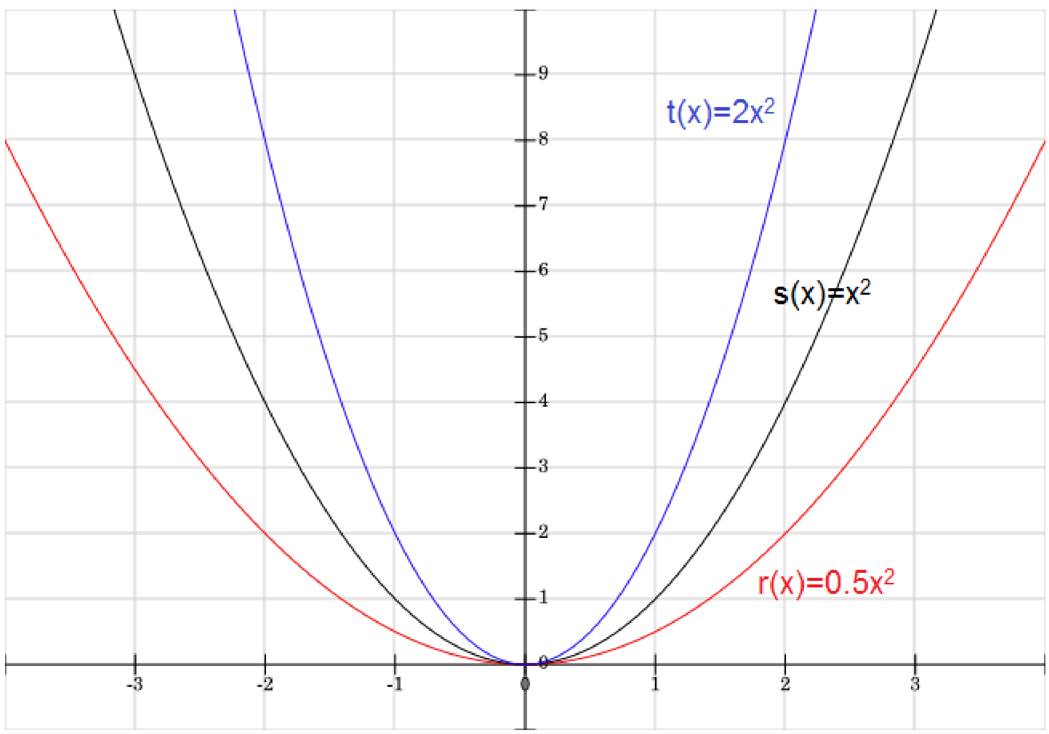

Figure 2: graphs of r(x), s(x) and t(x)

By the table and the graph, we can see this pattern: For $f(x)=ax^2$, as a's value increases, the parabola becomes "thinner"; as a's value decreases, the parabola becomes "wider".

Now, go ahead and graph $p(x)=-x^2$ and $q(x)=-2x^2$ on scratch paper. You will see the graph of $q(x)=-2x^2$ is thinner. However, from -1 to -2, the value of a actually decreased. This doesn't fit the pattern we observed!

The accurate way to state the pattern is:

For $f(x) = ax^2$, as a's **absolute value** increases, the parabola becomes "thinner"; as a's **absolute value** decreases, the parabola becomes "wider".

Again, don't try to memorize these patterns. Instead, on scratch paper, build a table, plot parabolas like what we did above, and re-discover these patterns as needed.