Putting Together Patterns of Parabola Movement

In this lesson, we will learn how to change a parabola's graph based on patterns we observed earlier. For the parabola $f(x)=a(x-h)^2+k$, we learned:

1. When $h>0$, the parabola shifts from the center to the right by h units; when $h<0$, the parabola shifts from the center to the left by h units.
2. When $k>0$, the parabola shifts up by k units; when $k<0$, the parabola shifts down by k units.
3. If $a>0$, the parabola faces up; if $a<0$, the parabola faces down.
4. When the absolute value of a increases, the parabola becomes thinner; when the absolute value of a decreases, the parabola becomes wider.

Note that the first two patterns can be boiled down to one: The vertex of $f(x)=a(x-h)^2+k$ is (h,k). Let's generalize the steps to sketch the graph of $f(x)=a(x-h)^2+k$:

1. Plot the vertex (h,k).
2. If $a>0$, the parabola faces up; if $a<0$, the parabola faces down.
3. If $|a|>1$, make the parabola thinner than the standard parabola $y=x^2$; if $|a|<1$, make the parabola wider than $y=x^2$; if $|a|=1$, leave the parabola as wide as $y=x^2$.

Notice the absolute value symbol in Step 3. This implies, when we graph $g(x)=-2x^2$, we should make the parabola thinner than $y=x^2$, because $|-2|>1$.

Don't worry too much about how "thinner" or "wider" your parabola should be. We will learn more tricks in later lessons to graph the parabola more accurately. For now, those 3 steps are good enough.

[**Example 1**] Without building a table, sketch the graph of $g(x)=2(x-1)^2-3$.

[**Solution 1**] We will use a "long way" in Solution 1, and then use a "short way" in Solution 2. Make sure both solutions make sense.

Let's start from the parabola of $f(x)=x^2$. We will keep track of the vertex. It is at $(0,0)$ at the beginning.

1. The part $(x-1)^2$ in $g(x)=2(x-1)^2-3$ implies we need to shift the parabola to the right by 1 unit. Now the vertex moved to $(1,0)$.

2. The part -3 in $g(x) = 2(x-1)^2 - 3$ implies we need to shift the parabola down by 3 units. Now the vertex moved to $(1,-3)$.
3. The 2 in $g(x) = 2(x-1)^2 - 3$ implies the parabola faces up, so no changes needs to be made.
4. Finally, the 2 in $g(x) = 2(x-1)^2 - 3$ implies we need to make the parabola "thinner". There is no need to show exactly how "thinner" the graph becomes. The vertex didn't move this time.

For this problem, the order of the steps above doesn't matter. We could have made the parabola thinner in step 1, shift down by 3 units, and finally shift right by 1 unit. We would have obtained the same graph for $g(x)$.

[**Solution 2**] Solution 2 is easier.

1. Identify the vertex of $g(x) = 2(x-1)^2 - 3$. It is $(1,-3)$.
2. Since $2 > 0$, the parabola faces up.
3. Since $|2| > 1$, the parabola should be thinner compared to the standard parabola $y = x^2$.

Both solutions generate the same graph for $g(x) = 2(x-1)^2 - 3$:

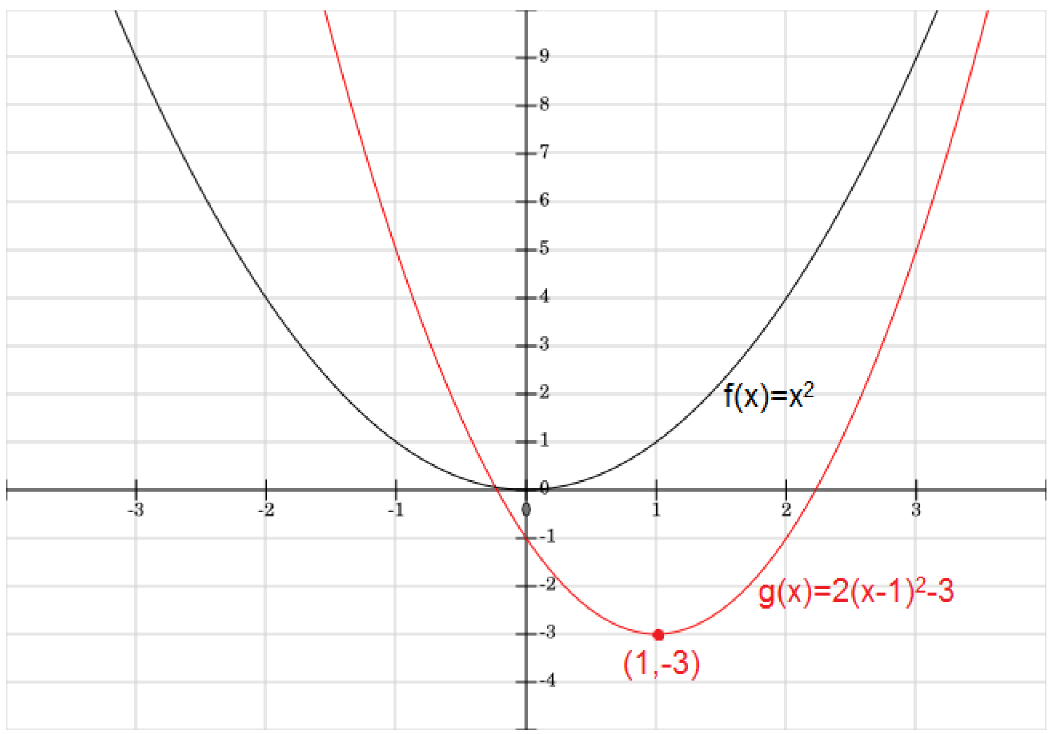

Figure 1: graphs of f(x)=x^2 and g(x)=$2(x-1)^2$-3

Let's look at another example.

[**Example 2**] Sketch the graph of $h(x) = -\frac{1}{2}(x+2)^2 - 1$.

[**Solution**] Let's use the "short way" from now on:

1. Identify the vertex $(-2,-1)$.
2. Since $-\frac{1}{2} < 0$, the parabola faces down.
3. Since $|-\frac{1}{2}| < 1$, the parabola is wider than the standard parabola $y = x^2$.

The graph looks like:

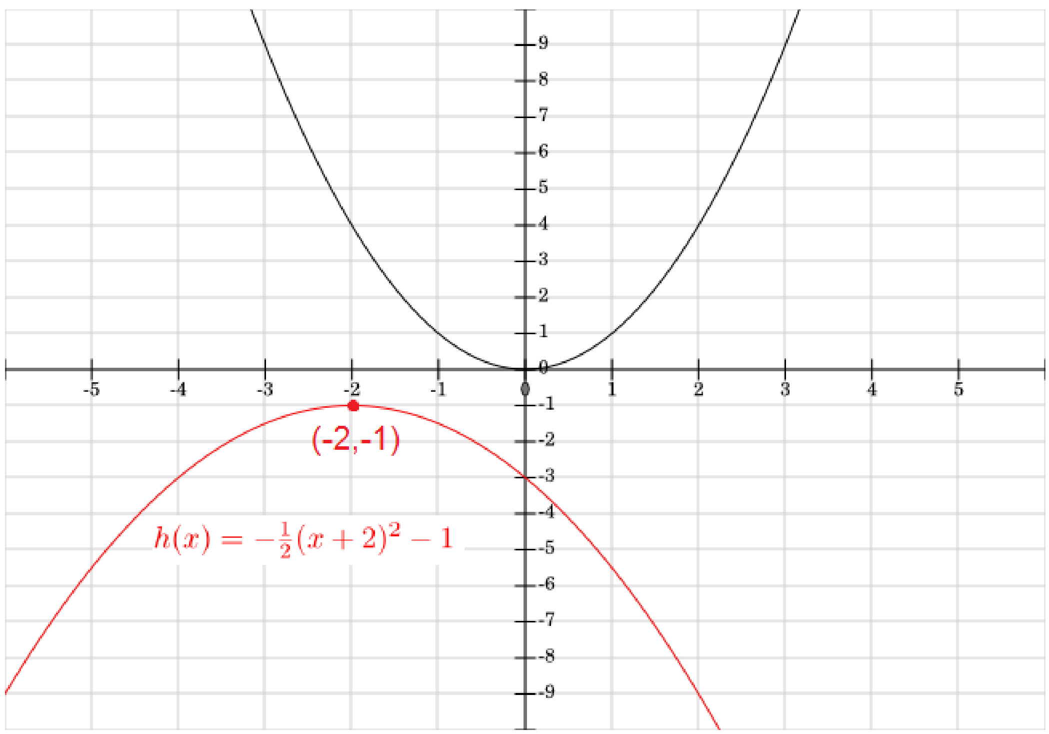

Figure 2: Graph of h(x)=(-1/2)(x+2)^2-1