Shifting Parabola Left/Right

Earlier, we learned that, for $f(x) = ax^2 + c$, changes in the value of c will shift the parabola up or down, and changes in the value of a will make the parabola thinner or wider.

Today, we will learn how to shift a parabola to the left or right. First, we need to learn two forms of a quadratic function.

Recall that a line's equation has different forms:

slope-intercept form: $y = Mx + B$

standard form: $Ax + By = C$

Similarly, a quadratic function has different forms:

standard form: $f(x) = ax^2 + bx + c$

vertex form: $f(x) = a(x-h)^2 + k$

For a line's equation, the slope-intercept form is more useful, telling us the line's slope M and *y*-intercept $(0, B)$.

For a quadratic function's equation, the vertex form is more useful, telling us the parabola's vertex (h, k), and the positive/negative sign of a tells us whether the parabola faces up or down. For example, we can tell the vertex of $f(x) = 2(x-1)^2 + 3$ is $(1,3)$. Its graph verifies this:

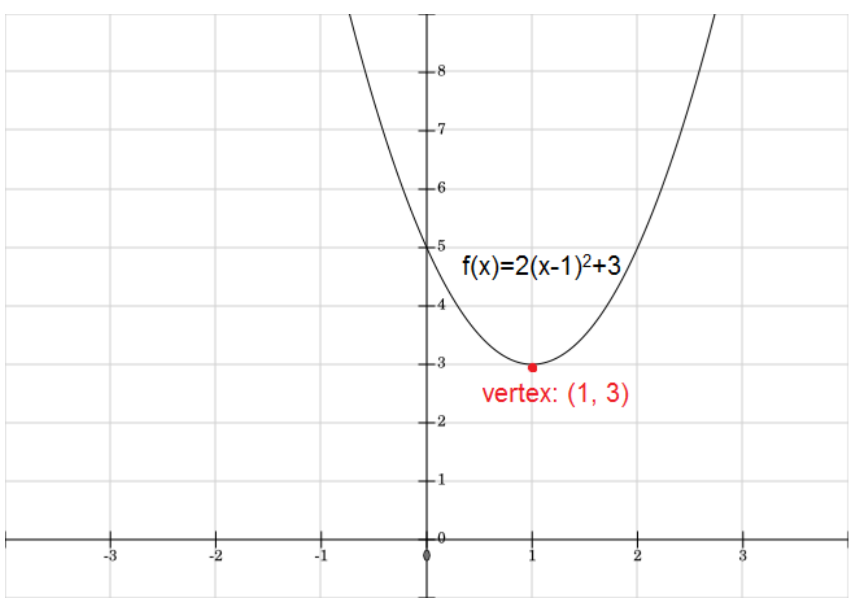

Figure 1: graph of f(x)=2(x-1)^2+3

In later lessons, we will learn how to change a quadratic function's equation from the less-useful standard form to the more-useful vertex form. In today's lesson, we will learn how the value of h changes a parabola's position in a graph.

Let's graph the following quadratic functions:

Table and graphs of $r(x)=(x-1)^2$, $s(x)=x^2$, $t(x)=(x+2)^2$

x	$r(x)=(x-1)^2$		x	$s(x)=x^2$		x	$t(x)=(x+2)^2$
-3	$y=(-3-1)^2=16$		-3	$y=(-3)^2=9$		-3	$y=(-3+2)^2=1$
-2	$y=(-2-1)^2=9$		-2	$y=(-2)^2=4$		-2	$y=(-2+2)^2=0$
-1	$y=(-1-1)^2=4$		-1	$y=(-1)^2=1$		-1	$y=(-1+2)^2=1$
0	$y=(0-1)^2=1$		0	$y=0^2=0$		0	$y=(0+2)^2=4$
1	$y=(1-1)^2=0$		1	$y=1^2=1$		1	$y=(1+2)^2=9$
2	$y=(2-1)^2=1$		2	$y=2^2=4$		2	$y=(2+2)^2=16$
3	$y=(3-1)^2=2$		3	$y=3^2=9$		3	$y=(3+2)^2=25$

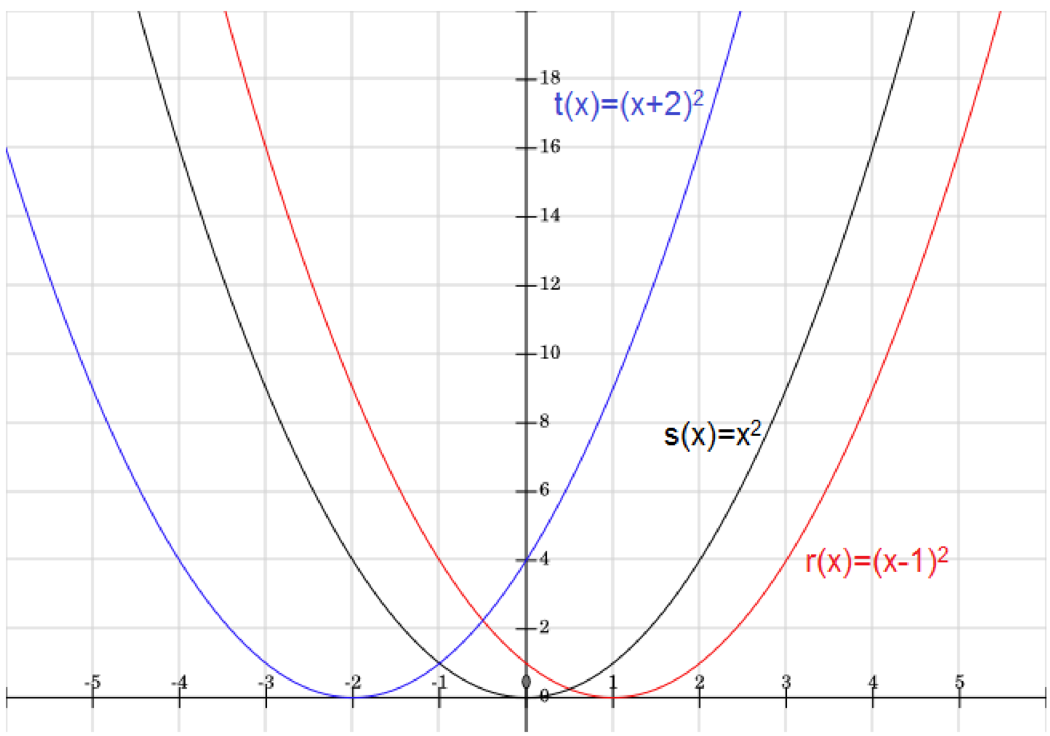

Figure 2: figures of f(x), g(x) and h(x)

Here is the pattern: Compared to $s(x)=x^2$, the graph of $r(x)=(x-1)^2$ shifted to the right by 1 unit, and the graph of $t(x)=(x+2)^2$ shifted to the left by 2 units.

This pattern is counter-intuitive, as the left is the negative direction! This is another reason why you shouldn't try to memorize these patterns. Build a table like what we did earlier, and you can observe the patterns.

To help you make sense why the graph of $t(x)=(x+2)^2$ shifted to the left, think this way:

For $s(x)=x^2$, to make $s(x)=0$, we can let $x=0$. The point $(0,0)$ is also the vertex of $s(x)=x^2$.

For $t(x)=(x+2)^2$, to make $t(x)=0$, we cannot plug in $x=0$ any more. We have to plug in $x=-2$. That's why the vertex of $t(x)=(x+2)^2$ is $(-2,0)$. Compared to $(0,0)$, the vertex shifted to the left.

I hope this helps you understand why the graph of $t(x)=(x+2)^2$ shifted to the left by 2 units, compare to the graph of $s(x)=x^2$.