Locate Axis and Vertex of Parabola

We learned that the vertex of $g(x) = a(x-h)^2 + k$ is (h,k). What if a quadratic equation is given in standard form $f(x) = ax^2 + bx + c$? In this lesson, we will learn how to locate the vertex for a standard-form quadratic equation.

Let me declare that I hate teaching math formulas without understanding. It makes students think math is about memorizing formulas. I hope you can tell by my lecture notes that I emphasize understanding. However, in this case, to help you understand the vertex formula, I have to use some Algebra skills which you will not need unless you will move on to later math courses. I decided to simply teach how to use the formula. Let's save the understanding part to the next math course.

Recall that the axis of a parabola is the vertical line crossing its vertex.

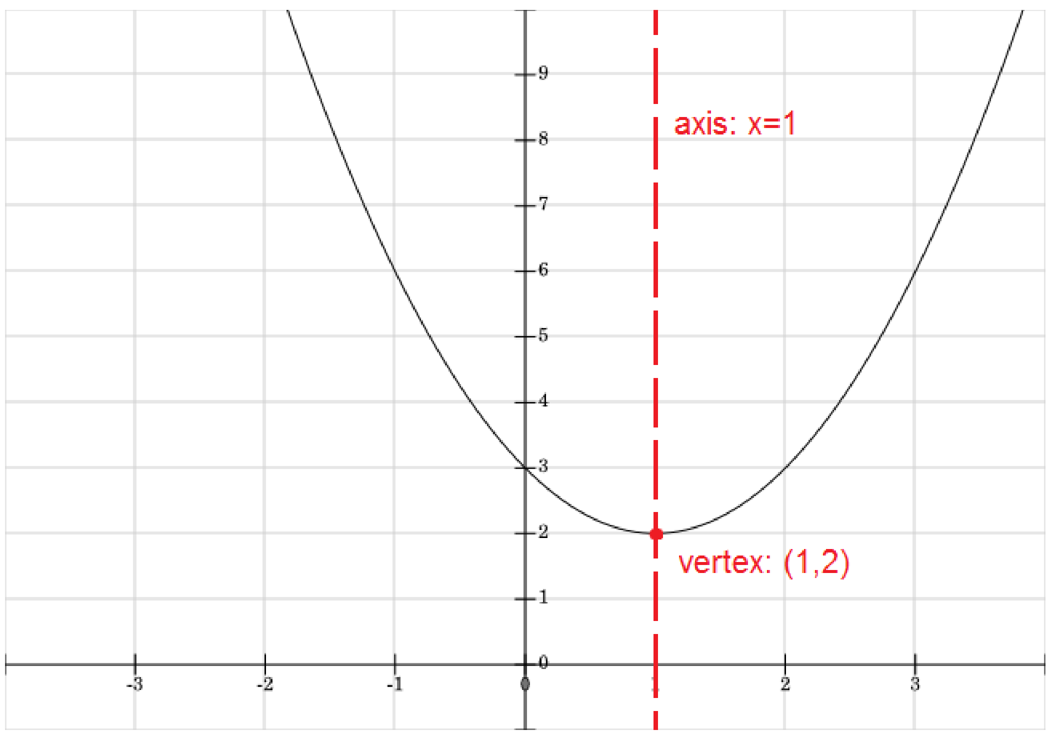

Figure 1: axis and vertex of a parabola

In this graph, the parabola's axis is $x = 1$, and its vertex is $(1,2)$. If its axis is $x = 3$, its vertex must be $(3, \text{a number})$, because a parabola's axis always crosses its vertex.

The axis formula and vertex formula are on the next page.

Here is the **axis formula**:

For $f(x) = ax^2 + bx + c$, its axis is $x = -\dfrac{b}{2a}$.

The **vertex formula** immediately follows the axis formula:

For $f(x) = ax^2 + bx + c$, its vertex is $(-\dfrac{b}{2a}, f(-\dfrac{b}{2a}))$.

The vertex formula is complicated. It should make more sense after we go through Example 1.

[**Example 1**] Find the axis and vertex of $h(x) = x^2 - 4x + 3$.

[**Solution**] First, identify that $a = 1, b = -4, c = 3$.

By the axis formula, $h(x)$'s axis is $x = -\dfrac{b}{2a} = -\dfrac{-4}{2 \cdot 1} = -(-2) = 2$.

Since the axis crosses the vertex, we know the vertex must be $(2, ?)$.

Remember how we build tables to graph a parabola? Say in a certain row, $x = 2$. How would you find the y value? In other words, when $x = 2$, what is $h(2)$?

We would plug $x = 2$ into the function, and we have:

$$h(2) = 2^2 - 4 \cdot 2 + 3 = -1$$

This is how we find the *y*-value of a quadratic function's vertex. Now the vertex formula $(-\dfrac{b}{2a}, f(-\dfrac{b}{2a}))$ should make sense. In Example 1, once we found that $-\dfrac{b}{2a} = 2$, the vertex formula became $(2, h(2))$.

Solution: The axis of $h(x)$ is $x = 2$, and the vertex of $h(x)$ is $(2, -1)$.

The graph of $h(x) = x^2 - 4x + 3$ is on the next page.

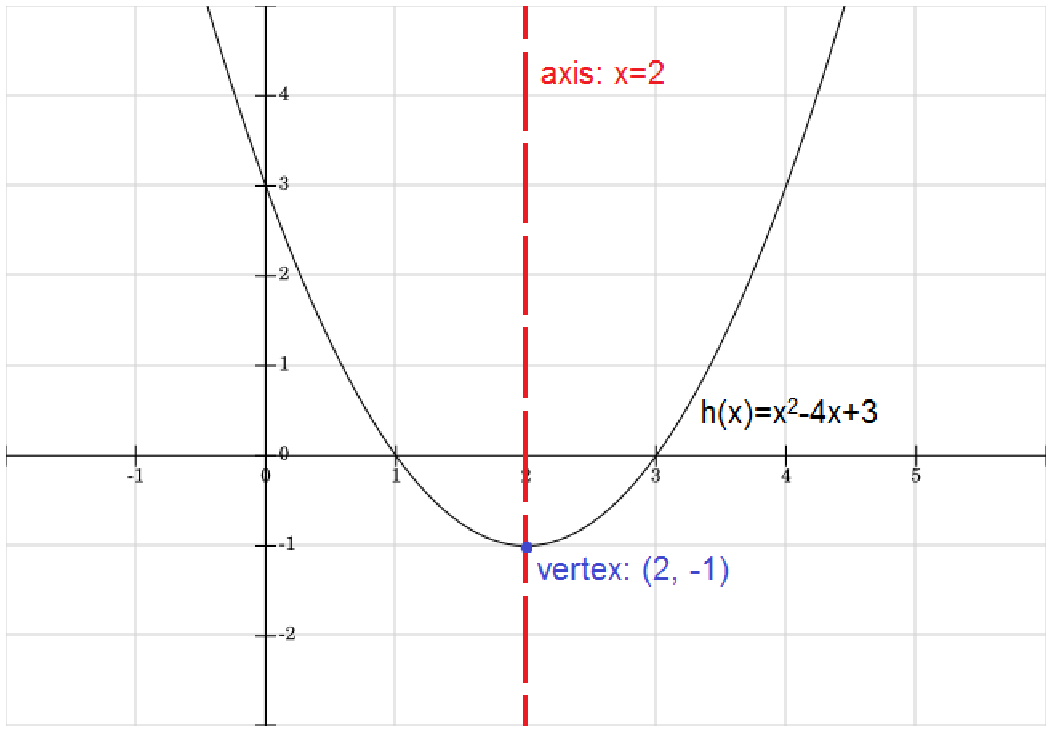

Figure 2: Graph of $h(x)=x^2-4x+3$

[**Example 2**] Find the axis and vertex of $p(x) = -6 - 4x - \frac{1}{2}x^2$

[**Solution**] First we need to change the function's equation into standard form:

$$p(x) = -\frac{1}{2}x^2 - 4x - 6$$

Next, identify that $a = -\frac{1}{2}, b = -4, c = -6$.

By the axis formula, the axis of $p(x)$ is $x = -\frac{b}{2a} = -\frac{-4}{2 \cdot (-\frac{1}{2})} = -\frac{-4}{-1} = -4$.

By the vertex formula, $p(x)$'s vertex is $(-4, p(-4))$. Plug $x = -4$ into $p(x)$, we have:

$$\begin{aligned} p(-4) &= -\frac{1}{2}(-4)^2 - 4(-4) - 6 \\ &= -\frac{1}{2}\cdot 16 - (-16) - 6 \\ &= -8 + 16 - 6 \\ &= 2 \end{aligned}$$

Solution: The axis of $p(x)$ is $x = -4$, and the vertex of $p(x)$ is $(-4,2)$.

Not all vertices (plural form of "vertex") have integer coordinates. You must be ready to handle fractions. See the next example.

[**Example 3**] Find the axis and vertex of $q(x) = \frac{1}{3}x^2 + x - 2$.

[**Solution**] Identify that $a = \frac{1}{3}, b = 1, c = -2$.

By the axis formula, the axis of $q(x)$ is $x = -\frac{b}{2a} = -\frac{1}{2\cdot\frac{1}{3}}$

There are many ways to get rid of the $\frac{1}{3}$ in the denominator. I will show two ways.

We can multiply 3 in both the numerator and denominator:

$$x = -\frac{b}{2a} = -\frac{1}{2\cdot\frac{1}{3}} = -\frac{1\cdot 3}{2\cdot\frac{1}{3}\cdot 3} = -\frac{3}{2}$$

Or, we can do fraction multiplication and then division:

$$x = -\frac{b}{2a} = -\frac{1}{2\cdot\frac{1}{3}} = -\frac{1}{\frac{2}{3}} = -1 \div \frac{2}{3} = -\frac{1}{1}\cdot\frac{3}{2} = -\frac{3}{2}$$

Now, for $q(x)$, the vertex formula becomes $(-\frac{3}{2}, q(-\frac{3}{2}))$. Let's calculate $q(-\frac{3}{2})$:

$$
\begin{aligned}
q(-\frac{3}{2}) &= \frac{1}{3}(-\frac{3}{2})^2 + (-\frac{3}{2}) - 2 \\
&= \frac{1}{3}\cdot\frac{9}{4} - \frac{3}{2} - 2 \\
&= \frac{3}{4} - \frac{3}{2} - 2 \\
&= \frac{3}{4} - \frac{6}{4} - \frac{8}{4} \\
&= -\frac{11}{4}
\end{aligned}
$$

Solution: The axis of $q(x)$ is $x = -\frac{3}{2}$, and the vertex of $q(x)$ is $(-\frac{3}{2}, -\frac{11}{4})$.