Factor Trinomial Part II

We learned how to factor trinomials like $x^2+7x+12$. Now let's learn how to factor trinomials like $2x^2-x-3$, where the leading term is not 1.

Guess and Check Method

The first method is to use either FOIL or area model to guess and check. I will show both models in this lecture. We start from an "empty shell":

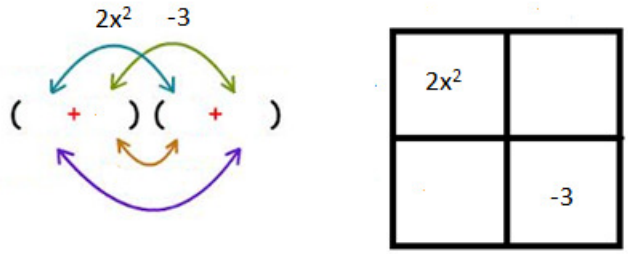

Figure 1: Ready to factor 2x^2-x-3

Next, think this way: What times what give $2x^2$? Well, $2x$ and x.

What times what give -3? It's either 3 and -1, or -3 and 1. We don't know which pair will work for this problem, so let's try 3 and -1 first. This is why this method is called "Guess and Check Method."

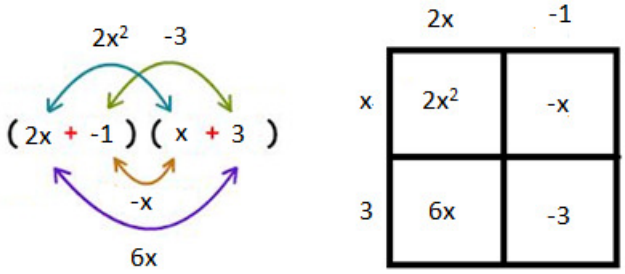

Figure 2: First try of factoring 2x^2-x-3

This model generates $(2x-1)(x+3)=2x^2+5x-3$, not what we want. Keep guessing. Let's switch places for -1 and 3:

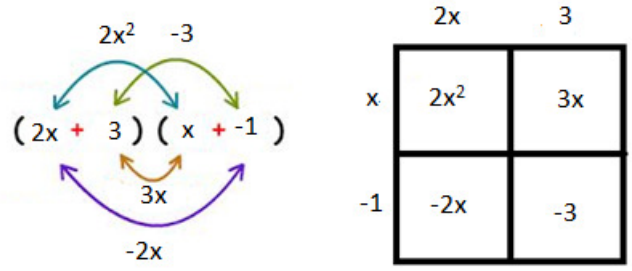

Figure 3: Second try of factoring 2x^2-x-3

This model generates $(2x+3)(x-1)=2x^2+x-3$, not what we want—we need $-x$, but the model generated $+x$. However, if the only difference is the positive or negative sign, we can simply switch the positive and negative signs in the graph, and get:

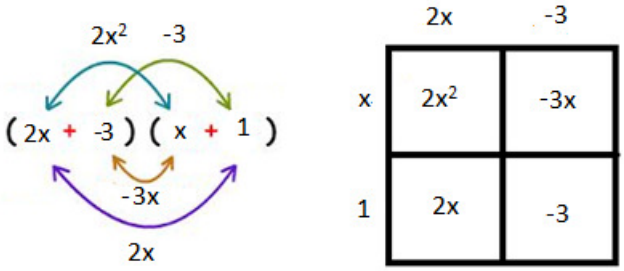

Figure 4: Third try to factor 2x^2-x-3

Finally, we have our solution: $2x^2-x-3=(2x-3)(x+1)$

The advantage of this method is that is makes sense, especially with the area model. We can see "base times height equals area", and that 4 pieces of area add up to the big rectangle's area, which equals $(2x-3)(x+1)$ in the example above.

The disadvantage of this method is that it takes time to guess and check. For example to factor $6x^2-5x-6$, we have to break $6x^2$ into $2x\cdot 3x$ and $6x\cdot x$, and break -6 into $6\cdot(-1)$, $-6\cdot 1$, $3\cdot(-2)$ and $-3\cdot 2$. In the worst case, we would guess and check 8 times before getting the right answer.

The "ac method" will save us time.

ac method

Let's try to factor the same problem $2x^2-x-3$.

Step 1: We treat $2x^2-x-3$ as ax^2+bx+c, and get $a=2$, $b=-1$ and $c=-3$.

Step 2: Do $ac = 2(-3) = -6$.

Step 3: List all possible two-number product of -6, we have:

$$1 \cdot (-6) = -6 \qquad -1 \cdot 6 = -6$$
$$2 \cdot (-3) = -6 \qquad -2 \cdot 3 = -6$$

Step 4: Find which pair's sum generates $b = -1$. It is 2 and -3.

Step 5: Change the middle term $-x$ into $2x - 3x$, and then do Factor by Grouping:

$$\begin{aligned} & 2x^2 - x - 3 \\ &= 2x^2 + 2x - 3x - 3 \\ &= 2x(x+1) - 3(x+1) \\ &= (2x-3)(x+1) \end{aligned}$$

Solution: $2x^2 - x - 3 = (2x-3)(x+1)$

The advantage of "ac method" is that it saves time.

The disadvantage of "ac method" is that it's a bunch of procedures which do not make a lot of sense. It makes people think math is a about memorizing procedures and formulas.

It's your choice which method to use. I personally prefer Guess and Check with the area model.

[**Example 1**] Factor $8x^2 - 4x - 12$

[**Solution**] If you start guess and check or ac method, you will waste your valuable time. The first step of factoring is to check whether you can factor out common factors. In this problem, we can factor out 4:

$$8x^2 - 4x - 12 = 4(2x^2 - x - 3)$$

Then you can use your method of choice to factor $2x^2 - x - 3$ as in the notes above, and we have:

$$8x^2 - 4x - 12 = 4(2x^2 - x - 3) = 4(2x-3)(x+1)$$

Solution: $8x^2 - 4x - 12 = 4(2x-3)(x+1)$

[**Example 2**] Factor $8x^2y - 4xy - 10y$

[**Solution**] Notice that we can factor out $2y$, and have

$$8x^2y - 4xy - 10y = 2y(4x^2 - 2x - 5)$$

Next, we will try to factor $4x^2 - 2x - 5$. Let's use the ac method.

Step 1: Identify that $a = 4, b = -2, c = -5$.

Step 2: $ac = 4(-5) = -20$

Step 3: List all 2-number product of -20:

$$\begin{aligned} &-1 \cdot 20 = -20 \qquad 1 \cdot (-20) = -20 \\ &-2 \cdot 10 = -20 \qquad 2 \cdot (-10) = -20 \\ &-3 \cdot \text{nothing} = -20 \\ &-4 \cdot 5 = -20 \qquad 4 \cdot (-5) = -20 \end{aligned}$$

Step 4: We cannot find any pair whose sum is -2. This implies $4x^2 - 2x - 5$ is prime.

Solution: $8x^2y - 4xy - 10y = 2y(4x^2 - 2x - 5)$

Note that we didn't fail to factor $8x^2y - 4xy - 10y$. We still turned subtraction into multiplication, except that the inside part, $4x^2 - 2x - 5$, cannot be further factored.

[**Example 3**] Factor $20x^4y - 38x^3y^2 + 12x^2y^3$

[**Solution**] Notice that we can factor out $2x^2y$, and we have:

$$20x^4y - 38x^3y^2 + 12x^2y^3 = 2x^2y(10x^2 - 19xy + 6y^2)$$

Next, let's use the ac method to factor $10x^2 - 19xy + 6y^2$.

Step 1: Identify that $a = 10, b = -19, c = 6$.

Step 2: $ac = 6 \cdot 10 = 60$

Step 3: List all 2-number product of 60:

$$1 \cdot 60 = 60 \qquad -1 \cdot (-60) = 60$$
$$2 \cdot 30 = 60 \qquad -2 \cdot (-30) = 60$$
$$3 \cdot 20 = 60 \qquad -3 \cdot (-20) = 60$$
$$4 \cdot 15 = 60 \qquad -4 \cdot (-15) = 60$$
$$\ldots$$

Step 4: We don't have to list more pairs because we already found the pair we need: -4 and -15.

Step 5: Now use Factor by Grouping:

$$\begin{aligned} & 10x^2 - 19xy + 6y^2 \\ &= 10x^2 - 4xy - 15xy + 6y^2 \\ &= 2x(5x - 2y) - 3y(5x - 2y) \\ &= (2x - 3y)(5x - 2y) \end{aligned}$$

Solution: $20x^4 y - 38x^3 y^2 + 12x^2 y^3 = 2x^2 y(2x - 3y)(5x - 2y)$