

Factor Trinomials Part 1

First, let's review.

The first step of factoring is to try to factor out common factors, like $2x - 4 = 2(x - 2)$.

If there are no common factors, and if there are 4 terms, try Factor by Grouping, like

$$\begin{aligned} & xy + x + 2y + 2 \\ &= (xy + x) + (2y + 2) \\ &= x(y + 1) + 2(y + 1) \\ &= (y + 1)(x + 2) \end{aligned}$$

Today we will learn how to factor polynomials like $x^2 + 7x + 12$.

Review Polynomial Multiplication

First, let's review how to multiply $(x + 3)(x + 4)$. We could use FOIL or the area model:

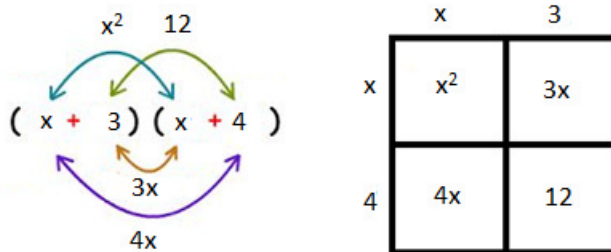


Figure 1: Multiply $(x+3)(x+4)$

We have: $(x + 3)(x + 4) = x^2 + 7x + 12$.

Factoring is the inverse operation of multiplying polynomials: $x^2 + 7x + 12 = (x + ?)(x + ?)$

Number Product/Sum Question

Factoring is more difficult than multiplying polynomials, because we have to guess and check. For this problem we need to ask ourselves:

Which two numbers' product is 12, and at the same time, their sum is 7?

Many of you can easily tell these two numbers are 3 and 4. However, let's learn a systematical way, so we can deal with bigger numbers later.

We need to list all possible ways to get a product of 12 with two numbers:

$$\begin{array}{ll}
 1 \cdot 12 = 12 & (-1) \cdot (-12) = 12 \\
 2 \cdot 6 = 12 & (-2) \cdot (-6) = 12 \\
 3 \cdot 4 = 12 & (-3) \cdot (-4) = 12
 \end{array}$$

Note that we started from 1 and stopped at 3. This is because the next number is 4, which is already considered in the third row.

Now, see which pair's sum is 7. We have the answer: 3 and 4.

That's how we can figure out $x^2 + 7x + 12 = (x + 3)(x + 4)$.

[Example 1] Factor $x^2 - 7x + 12$

[Solution] We already created a list of all possible product of 12. Looking through the list, -3 and -4 will have a sum of -7. The solution is $x^2 - 7x + 12 = (x - 3)(x - 4)$. We can check our solution by FOIL or area model:

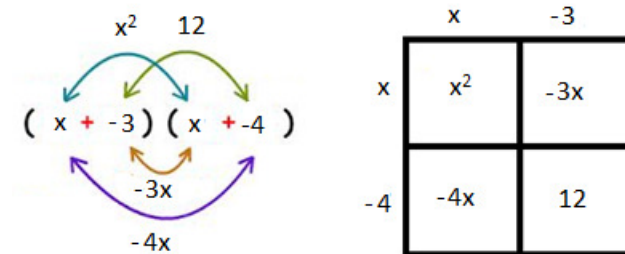


Figure 2: FOIL and area model for $(x-3)(x-4)$

[Example 2] Factor $x^2 - x - 72$

[Solution] We make a list of all possible two-number product of -72:

$$\begin{array}{ll}
 -1 \cdot 72 = -72 & 1 \cdot (-72) = -72 \\
 -2 \cdot 36 = -72 & 2 \cdot (-36) = -72 \\
 -3 \cdot 24 = -72 & 3 \cdot (-24) = -72 \\
 -4 \cdot 18 = -72 & 4 \cdot (-18) = -72 \\
 -5 \cdot (\text{nothing}) = -72 & \\
 -6 \cdot 12 = -72 & 6 \cdot (-12) = -72 \\
 -7 \cdot (\text{nothing}) = -72 & \\
 -8 \cdot 9 = -72 & 8 \cdot (-9) = -72
 \end{array}$$

Note that we considered each number from 1 all the way up to 8, including 5 and 7, which do not go into 72. This is how you don't miss any number. Don't think randomly, jumping from $1 \cdot 72$ to $4 \cdot 18$.

We stopped at 8 because the next number is 9, which is already considered in the line

$$-8 \cdot 9 = -72 \quad 8 \cdot (-9) = -72.$$

We need to find a pair whose sum is -1, and the pair is 8 and -9.

$$\text{Solution: } x^2 - x - 72 = (x + 8)(x - 9)$$

"ac method"

In case some of you learned the "ac method" in high school, and would love to use it, here is how to do

$x^2 - x - 72$ in "ac method".

You still have to list all possible 2-number product of -72, and find that 8 and -9 is the pair you need.

Next, do this:

$$\begin{aligned} x^2 - x - 72 \\ &= x^2 + 8x - 9x - 72 \\ &= x(x + 8) - 9(x + 8) \\ &= (x + 8)(x - 9) \end{aligned}$$

You might wonder why go through these extra steps involving Factor by Grouping. Later, we will learn how to factor trinomials like $2x^2 - x - 3$. When the leading coefficient is not 1, "ac method" has an advantage.

[Example 3] Factor $2x^3 - 2x^2 - 144x$

[Solution] You will be stuck if you forget that the first step of factoring is to try to factor out common factors. For this problem, the common factor is $2x$, which goes into all 3 terms. So we have:

$$2x^3 - 2x^2 - 144x = 2x(x^2 - x - 72)$$

Next, we factor $x^2 - x - 72$ like what we did in Example 2, and we have:

$$2x^3 - 2x^2 - 144x = 2x(x^2 - x - 72) = 2x(x + 8)(x - 9)$$

It's a common mistake to forget $2x$ in the final answer.

There is one more example on the next page.

[Example 4] Factor $x^2 - xy - 72y^2$

[Solution] We cannot factor out any common factors. Don't be afraid of a second variable. The FOIL and area model still work:

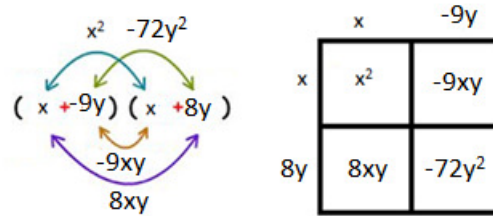


Figure 3: FOIL or area model to factor $x^2 - xy - 72y^2$

Solution: $x^2 - xy - 72y^2 = (x - 9y)(x + 8y)$