Factor by Grouping

Let's review how to factor our common factors:

$$xy + 2y = y(x + 2)$$

In this problem, *y* can be factored out because it is common in both terms. What if *y* is a binomial?

$$x(y + 1) + 2(y + 1) = (y + 1)(x + 2)$$

This still works, if we treat $y + 1$ as a common factor.

Factor by Grouping has one more step than what we just did.

[**Example 1**] Factor $xy + x + 2y + 2$.

[**Solution**] Whenever you see 4 terms, and if you cannot combine any, we need to try the method of Factor by Grouping.

1. Break those 4 terms into two groups, each with 2 terms with some common factor. For this example, we have $xy + x + 2y + 2 = (xy + x) + (2y + 2)$.
2. Factor out the common factor in each group, we have:
   $(xy + x) + (2y + 2) = x(y + 1) + 2(y + 1)$.
3. Factor out the common factor from both groups, we have:
   $x(y + 1) + 2(y + 1) = (y + 1)(x + 2)$.

Put together these 3 steps, we have:

$$\begin{aligned} & xy + x + 2y + 2 \\ &= (xy + x) + (2y + 2) \\ &= x(y + 1) + 2(y + 1) \\ &= (y + 1)(x + 2) \end{aligned}$$

Be careful when negative signs are involved, as in Example 2.

[**Example 2**] Factor $xy + x - 2y - 2$

[**Solution**] Following those 3 steps of Factor by Grouping, we have:

$$\begin{aligned} & xy + x - 2y - 2 \\ &= (xy + x) + (-2y - 2) \\ &= x(y + 1) - 2(y + 1) \\ &= (y + 1)(x - 2) \end{aligned}$$

Note that $-2y - 2 = -2(y + 1)$. It a common mistake to write $-2y - 2 = -2(y - 1)$.

[**Example 3**] Factor $x^3 - 3x^2 - 5x + 15$

[**Solution**]

$$\begin{aligned} & x^3 - 3x^2 - 5x + 15 \\ &= (x^3 - 3x^2) + (-5x + 15) \\ &= x^2(x - 3) - 5(x - 3) \\ &= (x^2 - 5)(x - 3) \end{aligned}$$

Note all polynomials can be factored. See Example 4.

[**Example 4**] Factor $\mathrm{x}^3 - 3\mathrm{x}^2 - 5\mathrm{x} + 10$

[**Solution**]

$$\begin{aligned} & x^3 - 3x^2 - 5x + 10 \\ &= (x^3 - 3x^2) + (-5x + 10) \\ &= x^2(x - 3) - 5(x - 2) \end{aligned}$$

This is as far as we can go. Since we didn't turn addition/subtraction into multiplication, this polynomial cannot be factored. We call such a polynomial "prime", just like the prime number 5, which cannot be factored.

Conclusion: $\mathrm{x}^3 - 3\mathrm{x}^2 - 5\mathrm{x} + 10$ is prime.

Don't forget what we learned earlier: We should factor out common factors whenever possible.

[**Example 5**] Factor $x^4y-3x^3y-5x^2y+15xy$

[**Solution**]

$$\begin{aligned}
&x^4y-3x^3y-5x^2y+15xy\\
&=xy(x^3-3x^2-5x+15)\\
&=xy[(x^3-3x^2)+(-5x+15)]\\
&=xy[x^2(x-3)-5(x-3)]\\
&=xy(x^2-5)(x-3)
\end{aligned}$$

We would get the same solution if we "group" first:

$$\begin{aligned}
&x^4y-3x^3y-5x^2y+15xy\\
&=(x^4y-3x^3y)+(-5x^2y+15xy)\\
&=x^3y(x-3)-5xy(x-3)\\
&=(x-3)(x^3y-5xy)\\
&=(x-3)xy(x^2-5)\\
&=xy(x-3)(x^2-5)
\end{aligned}$$

However, it's in general easier if we factor out common factors first.