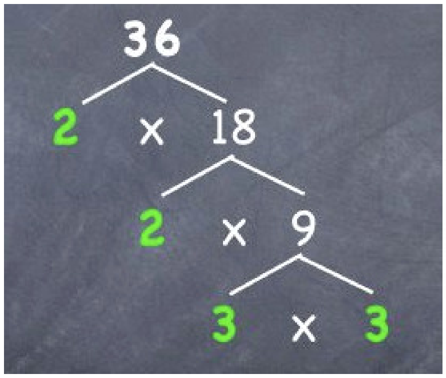
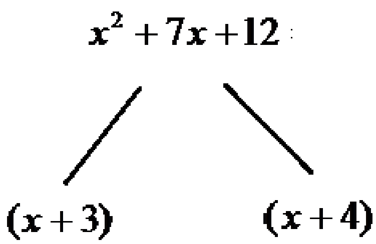
First Step of Factoring

Factoring breaks down a number to its prime factors. Here are two examples:



$$36 = 2^2 \cdot 3^2$$

Figure 1: Factoring 36



$$x^2 + 7x + 12 = (x+3)(x+4)$$

Figure 2: Factoring x^2+7x+12

Motivation of Factoring

Why do we need to factor a number or a polynomial?

There are many reasons. I will write one reason here. Let's review how to reduce a fraction.

$$\frac{2}{4} = \frac{2 \cdot 1}{2 \cdot 2} = \frac{\cancel{2} \cdot 1}{\cancel{2} \cdot 2} = \frac{1}{2}$$

When we reduce a fraction, we can cross out the same number when both the numerator and the denominator are factored.

Note that you may **NOT** cross out the same number when the operation is addition/subtraction, like this:

$$\frac{3}{4} = \frac{2+1}{2+2} = \frac{\cancel{2}+1}{\cancel{2}+2} = \frac{1}{2}$$

This makes no sense as $\frac{3}{4}$ and $\frac{1}{2}$ are not equivalent.

How to reduce this fraction: $\frac{x+3}{x^2+7x+12}$?

We may not cross out any number yet because the operation is addition. However, we can factor first and then reduce it:

$$\frac{x+3}{x^2+7x+12}=\frac{(x+3)\cdot 1}{(x+3)\cdot(x+4)}=\frac{\cancel{(x+3)}\cdot 1}{\cancel{(x+3)}\cdot(x+4)}=\frac{1}{x+4}$$

We can cross out $(x+3)$ after we changed addition to multiplication by factoring.

This reduction implies $\frac{x+3}{x^2+7x+12}$ and $\frac{1}{x+4}$ are equivalent (not exactly, but I don't want to confuse you now). Really? Let's plug in *x*=1 and check. When *x*=1, we have:

$$\frac{x+3}{x^2+7x+12}=\frac{1+3}{1^2+7\cdot 1+12}=\frac{4}{20}=\frac{1}{5}\text{, and}$$

$$\frac{1}{x+4}=\frac{1}{1+4}=\frac{1}{5}$$

It works! Actually, no matter what number you plug into *x*, $\frac{x+3}{x^2+7x+12}$ and $\frac{1}{x+4}$ are always equivalent. (Again, not exactly, but this is good enough for now.)

First Step of Factoring

When you have a polynomial like this one:

$$25x^5+75x^4-125x^3$$

the first step is to factor out the common terms, like in this number example:

$$12+18=2^2\cdot 3+2\cdot 3^2=2\cdot 3\cdot(2+3)$$

Note that we factored out $2\cdot 3$, or 6, as 6 goes into both 12 and 18. We may **NOT** simply factor out 2 and be done, like this:

$$12+18=2^2\cdot 3+2\cdot 3^2=2\cdot(6+9)$$

We have to factor out as many factors as possible.

[**Example 1**] Factor $25x^5 + 75x^4 - 125x^3$

[**Solution**] When we factor out common terms, we follow these steps:

1. For numbers, 25 goes into all 3 terms; for variables, x^3 goes into all 3 terms. Together, we can factor out $25x^3$.
2. Rewrite the polynomial into $25x^5 + 75x^4 - 125x^3 = 25x^3(\_\_\_ + \_\_\_ + \_\_\_)$.
3. For the first term, think: $25x^3$ times what brings $25x^5$ back? The answer is x^2, so we have $25x^5 + 75x^4 - 125x^3 = 25x^3(x^2 + \_\_\_ + \_\_\_)$.
4. For the second term, think: $25x^3$ times what brings $75x^4$ back? The answer is $3x$, so we have $25x^5 + 75x^4 - 125x^3 = 25x^3(x^2 + 3x + \_\_\_)$.
5. For the third term, think: $25x^3$ times what brings $-125x^3$ back? The answer is -5, so we have $25x^5 + 75x^4 - 125x^3 = 25x^3(x^2 + 3x - 5)$.

Solution: $25x^5 + 75x^4 - 125x^3 = 25x^3(x^2 + 3x - 5)$.

[**Example 2**] Factor $-63x^2y^6 - 54xy^7 + 81y^8$

[**Solution**] When the leading term is negative, usually we factor out the negative sign to make our life easier.

1. For numbers, we can factor out -9; for variables, we can factor out y^6. Together, we can factor out $-9y^6$.
2. We have: $-9y^6(\_\_\_ + \_\_\_ + \_\_\_)$.
3. $-9y^6$ times what brings $-63x^2y^6$ back? The answer is $7x^2$. So we have $-9y^6(7x^2 + \_\_\_ + \_\_\_)$.
4. $-9y^6$ times what brings $-54xy^7$ back? The answer is $6xy$. So we have $-9y^6(7x^2 + 6xy + \_\_\_)$.

5. $-9y^6$ times what brings $81y^8$ back? The answer is $-9y^2$. So we have $-9y^6(7x^2+6xy-9y^2)$.

Solution: $-63x^2y^6-54xy^7+81y^8=-9y^6(7x^2+6xy-9y^2)$.

[**Example 3**] Factor $x-2$

[**Solution**] There is no common factor to factor out. We cannot factor this polynomial. We call such a polynomial "prime", like the prime number 5, which cannot be further factored.

Later, as we learn more and more methods to factor, we tend to forget that the first step of factoring is to factor out common terms. Try not to!